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Sampling procedures for inspection by attributes — Two-stage sampling plans for auditing and for inspection under prior information

Règles d'échantillonnage pour l'inspection par attributs — Plans d'échantillonnage à deux niveaux pour l'audit et l'inspection des lots en exploitant l'information a priori

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Contents

Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 69, *Application of statistical methods*, Subcommittee SC 5, *Acceptance sampling*.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document addresses several application domains: financial auditing, lot inspection, quality auditing, functional testing, conformance inspection and acceptance testing. In all these domains, users are concerned with the decision problem of accepting or rejecting an inspection target.

The two-stage sampling scheme suggested by this document addresses three areas of inspection practice:

a) adjust sample sizes to prior information on the status of the inspection target;

b) enable a rapid decision by samples of small size if the population submitted for inspection is actually in very good or very bad condition, and enforce higher sample sizes only if the population submitted for inspection is actually in a medium condition;

c) protect against both errors of

   1) erroneously rejecting a tolerable inspection target, and

   2) erroneously accepting an intolerable inspection target.

To satisfy a), the sampling plans in this document are indexed in the parameter Trust with levels low, mid, high, where increasing Trust level reduces sample size. To satisfy b), this document imposes two-stage sampling plans with small sample sizes in the first stage and higher sample sizes in the second stage, where ordinarily a decision is reached already in the first stage if the population submitted for inspection is somewhere in-between.

The sampling scheme in this document is particularly suitable for financial auditing, both for auditing the internal control system (ICS) and for usage in tests of details as a tool of substantive procedures in financial auditing. ICS auditing and test of details are usually based on sampling instead of screening procedures. The relevant standard ISA 530 requires that sampling enable conclusions on the full population. Conclusively, statistical sampling schemes are indispensable.

Previous inspection results will be an important basis for the choice of the trust level for later inspections. Thus, the continued use of the sampling scheme in this standard will serve as an incentive for the providers of the respective targets, e.g. the responsible authorities for the ICS in a company, to improve upon the quality of the target populations.

The decision procedure of the sample is kept simple for immediate implementation. In particular, the user is not requested to evaluate mathematical formulae.

The target population is considered as acceptable (tolerable) if the proportion nonconforming does not exceed a specified tolerance p_0, otherwise it is considered as unacceptable (intolerable). Correspondingly, the objective of sampling inspection is to enable a decision between the alternatives of "acceptance" and "rejection". In different application domains, acceptance and rejection have different practical interpretations, see the explanations in Clause 5.

The sampling inspection procedure starts with a first sample of size n_1 with the following rule: accept if and only if no nonconforming units are found among the n_1 sampled units; reject if and only if at least Re_1 (stage 1 rejection number) nonconforming units are found among the n_1 sampled units; proceed to the second stage if and only if at least one and at most Re_1-1 nonconforming units are found among the n_1 sampled units. In the second stage, sample n_2 units, and decide "accept" if and only if the number of nonconforming units in the combined first and second sample is smaller or equal to the stage 2 acceptance number Ac_2, otherwise reject. The two-stage decision procedure can be expressed equivalently by comparing the limits of a two-sided confidence interval of nominal level (γ) for the proportion nonconforming with the tolerance p_0.

The sampling plans are indexed by three quantities:

i) the tolerance p_0;

ii) the nominal confidence level (γ), which is respectively either 0,7, 0,8, 0,9, 0,95 or 0,99;

iii) three levels, low, mid, high, of a scale called Trust.

The Trust levels express the user's degree of confidence into the status of the target population.

The objective of this document is to provide procedures that enable a decision quickly and economically if the proportion nonconforming is particularly low or high. In the latter case, the inspection procedure will in most all cases terminate in stage 1 with small sample sizes n_1. Only under intermediate values of the proportion nonconforming in the target population, the likelihood of proceeding to a second sample is high. The two sample sizes in stage 1 and stage 2 are chosen so as to minimize the expected sample size under the specified confidence level and Trust level.

Sampling procedures for inspection by attributes — Two-stage sampling plans for auditing and for inspection under prior information

1 Scope

This document specifies two-stage (double) sampling plans by attributes for inspection for a proportion of nonconforming items in a target population of discrete units, in particular:

a) the proportion of nonconforming items in a lot of product items;

b) the proportion of nonconforming function instances of an internal control system (ICS);

c) the proportion of misstatements in a population of accounting entries or booking records;

d) the proportion of nonconforming test characteristics of an entity subject to an acceptance test, e.g. in product and process audits.

The plans are preferable to single sampling plans where the cost of inspection is high or where the delay and uncertainty caused by the possible requirement for second samples is inconsequential. The statistical theory underlying the plans, tables and figures are provided in Annexes A through K.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2859-2, *Sampling procedures for inspection by attributes — Part 2: Sampling plans indexed by limiting quality (LQ) for isolated lot inspection*

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms, definitions, symbols and abbreviated terms

3.1 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 2859-2, ISO 3534-1 and ISO 3534-2 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1.1
acceptance number
Ac
largest number of nonconformities or nonconforming items found in the sample by *acceptance sampling* (3.1.2) by attributes that permits the acceptance of the lot, as given in the *acceptance sampling plan* (3.1.3)

[SOURCE: ISO 3534-2:2006, 4.4.2]

3.1.2
acceptance sampling
sampling after which decisions are made to accept or not to accept a lot, or other grouping of products, materials or services, based on sample results

[SOURCE: ISO 3534-2:2006, 1.3.17]

3.1.3
acceptance sampling plan
plan which states the sample size(s) to be used and the associated criteria for lot acceptance

[SOURCE: ISO 3534-2:2006, 4.3.3]

3.1.4
Conditional risks

3.1.4.1
conditional risk type I
conditional type I
conditional probability that $H_0: p \leq p_0$ be accepted by the test, given that $p > p_0$

Note 1 to entry: The conditional type I error is also addressed as the conditional type I risk.

Note 2 to entry: In an auditing context, the conditional type I risk evaluates the extent in which the purpose of auditing is failed. Hence, the type I risk can also be considered as a conditional measure of audit effectiveness.

Note 3 to entry: For a detailed mathematical explanation of this conditional risk, see Annex D.

3.1.4.2
conditional risk type II
conditional type II
conditional probability that $H_0: p \leq p_0$ be not accepted by the test, given that $p \leq p_0$

Note 1 to entry: The conditional type II error is also addressed as the conditional type II risk.

Note 2 to entry: In an auditing context, good lots are rejected in this case. So, the conditional type II risk is a measure for economic loss. Hence, the type II risk can also be considered as a conditional measure of auditing efficiency.

Note 3 to entry: For a detailed mathematical explanation of this conditional risk, see Annex D.

3.1.5
confidence interval
interval calculated from the sample, which specifies a range of plausible values of the unknown parameter *p*

Note 1 to entry: The reliability of the confidence interval as an interval estimate for *p* is measured by the actual coverage probability, i.e. the probability that the interval contain the true value of *p*. For a confidence interval of nominal level γ, the actual coverage probability has the lower bound γ pointwise in *p*. The length of the confidence interval corresponds to the precision of the statistical inference on *p*. Thus, interest is in shortest confidence intervals.

Note 2 to entry: See Annex A

3.1.6
coverage probability
probability that a random confidence region contain the true value of *p*

Note 1 to entry: For a detailed mathematical explanation of the coverage probability, see Annex G.

3.1.7
financial statement
formal record that reports about an entity's financial activities and position, related to one point in time or to changes within a period in time

3.1.8
inspection by attributes
inspection by noting the presence, or absence, of one or more particular characteristic(s) in each of the items in the group under consideration, and counting how many items do, or do not, possess the characteristic(s), or how many such events occur in the item, group or opportunity space

[SOURCE: ISO 3534-2:2006, 4.1.3]

3.1.9
integrated average sample number
I.ASN
number measuring the average sample size resulting from a sampling plan under a given proportion nonconforming *p*, weighted according to prior information on *p*

Note 1 to entry: For a detailed mathematical explanation of *I.ASN*, see Annex F.

Note 2 to entry: If costs for sampling single units are given, the *I.ASN* can be used to estimate average sampling costs of the two-stage plan $(n_1;n_2)$.

3.1.10
integrated second stage probability
$I.p_{2nd}$
probability of requiring the second step

Note 1 to entry: For a detailed mathematical explanation of $I.p_{2nd}$, see Annex E.

3.1.11
lot
definite part of a population constituted under essentially the same conditions as the population with respect to the sampling purpose

[SOURCE: ISO 3534-2:2006, 1.2.4]

3.1.12
misstatement
difference between the required amount, classification, presentation or disclosure of a financial statement and the actual observed one

3.1.13
nonconforming item
nonconforming unit
item or unit with one or more nonconformities

[SOURCE: ISO 3534-2:2006, 1.2.12, modified — "unit" has been added to "item".]

3.1.14
nonconformity
non-fulfilment of a requirement

[SOURCE: ISO 3534-2:2006, 3.1.11]

3.1.15
operating characteristic
OC
probability of reaching the decision "acceptance" by a sampling plan, considered as a function of the true value of the proportion nonconforming *p*

Note 1 to entry: See Annex B.

3.1.16
OC matched
sampling plans that have the same operating characteristic

Note 1 to entry: See Annex C.

3.1.17
prior information
knowledge about a parameter before the actual sampling evidence is taken into account

Note 1 to entry: Sources of prior knowledge are, for instance, historic audits and the assessment of the company environment.

3.1.18
population
totality of items under consideration

[SOURCE: ISO 3534-2:2006, 1.2.1]

3.1.19
rejection number
Re
smallest number of nonconformities or nonconforming items found in the sample by acceptance sampling by attributes that requires the lot to be not accepted, as given in the acceptance sampling plan

[SOURCE: ISO 3534-2:2006, 4.4.1]

3.1.20
sample
subset of a population made up of one or more sampling units

[SOURCE: ISO 3534-2:2006, 1.2.17, modified — Note 1 to entry deleted.]

3.1.21
substantive procedure
audit procedure with the objective of detecting misstatements at the assertion level

Note 1 to entry: There are two types of substantive procedures:

a) tests of details (of classes of transactions, account balances, and disclosures); and

b) substantive analytical procedures.

3.1.22
test of controls
audit procedure with the objective of assessing the operating effectiveness of controls in preventing, or detecting and correcting, material misstatements at the assertion level

3.1.23
tolerance proportion
largest value p_0 of the proportion nonconforming such that the target population is considered as acceptable

3.2 Symbols and abbreviated terms

n_i	sample sizes in stage *i*
n_{match}	one stage sample size with same OC as two stage sampling plan
x_i	number of misstated items (nonconforming items) found in n_i
D	confidence interval for the proportion of misstatements (nonconforming items)
p	proportion of misstatements (nonconforming units)
p_L	lower limit of *D*
p_U	upper limit of *D*
p_0	tolerance proportion
I.cp	integrated actual coverage
γ	nominal confidence level
a, b	shape parameters of the beta distribution
Ac_i	acceptance number in stage *i*
Re_i	rejection number in stage *i*
c.type I	conditional probability of erroneous acceptance
c.type II	conditional probability of erroneous rejection
$I.p_{2nd}$	integrated probability of entering the second stage
I.ASN	integrated average sample number
N	lot size
OC	operating characteristic function
P_a	probability of acceptance (OC function at a specified value *p*)

4 Selecting and operating a two-stage sampling plan under prior information

4.1 General

Table 1 to Table 5 in Clause 7 provide two-stage sampling plans $(n_1,(\mathrm{Ac}_1;\mathrm{Re}_1),n_2,(\mathrm{Ac}_2;\mathrm{Re}_2))$ indexed in the parameters p_0 (tolerance proportion), γ (confidence level), and in the level of prior information (trust).

The aim of the application of a two-stage sampling plan is two-fold:

a) enable a decision on whether or not the actual proportion nonconforming *p* exceeds the tolerance proportion p_0. In statistical terminology, the decision problem can be considered as a test of the hypothesis $H: p \leq p_0$ versus the alternative $K: p > p_0$;

b) provide a confidence interval for the actual proportion nonconforming p.

The design of the sampling plans assures that the probabilities of both decision errors 1) erroneous rejection of *H*, and 2) erroneous acceptance of *H* are bounded.

4.2 Selecting a sampling plan

Sampling plans can be obtained from Table 1 to Table 5 in Clause 7. The cell entries Table 1 to Table 5 display:

a) upper left: n_1 sample size in stage 1;

b) upper right: $(\mathrm{Ac}_1;\mathrm{Re}_1)$ acceptance and rejection number in stage 1;

c) lower left: n_2 sample size in stage 2;

d) lower right: $(\mathrm{Ac}_2;\mathrm{Re}_2)$ acceptance and rejection number in stage 2.

The sampling plans are indexed in p_0 (tolerance proportion), γ (nominal confidence level), and in the Trust level.

The nominal confidence level γ determines the reliability of the conclusive decision as taken according to the algorithm in 5.3 in the sense that the coverage probability $P(p\in D)$ exceeds γ for a wide range of actual values p, except a small interval around p_0, see the coverage probability graphs in Figures G.1 to G.5. Correspondingly, the probabilities of both decision errors 1) erroneous rejection of $H:p\leq p_0$, and 2) erroneous acceptance of $H:p\leq p_0$ are bounded by 1-γ for a wide range of actual values p.

The level of prior information shall be specified on an ordinal scale named Trust, by choosing among the values {low, mid, high}. The Trust level low shall be used if no prior experience or bad prior experience with populations submitted for inspection exists. The Trust level high shall be used if there is strong evidence of good performance. The Trust level mid shall be used if there is weak evidence of good performance or strong evidence of in-between performance.

See Annex H for further technical background on the prior information model and the Trust scale.

4.3 Sampling and decision procedure

The decision by a two-stage sampling plan $(n_1,(\mathrm{Ac}_1;\mathrm{Re}_1),n_2,(\mathrm{Ac}_2;\mathrm{Re}_2))$ shall proceed according to the following algorithm with $\mathrm{Ac}_1 = 0$:

Stage 1:

Draw a random sample of size n_1, determine the number x_1 of nonconforming units among the n_1 sampled units. Decide according to the subsequent cases a), b), and c):

a) $x_1 \leq \mathrm{Ac}_1$: Acceptance of the hypothesis $H:p\leq p_0$, i.e. p is considered not to exceed the tolerance p_0;

b) $x_1 \geq \mathrm{Re}_2$: Rejection of the hypothesis $H:p\leq p_0$, i.e. p is considered to exceed the tolerance p_0;

c) $\mathrm{Ac}_1 < x_1 < \mathrm{Re}_1$: No decision in stage 1, go to sample stage 2.

Stage 2:

If, in stage 1, the case c) occurs and enforces entering stage 2, proceed as follows:

Draw a second random sample of size n_2, determine the number x_2 of nonconforming units among the n_2 sampled units. Decide according to the subsequent cases a) and b):

a) $x_1 + x_2 \leq \mathrm{Ac}_2$: Acceptance of the hypothesis $H:p\leq p_0$, i.e. p is considered not to exceed the tolerance p_0;

b) $x_1 + x_2 \geq \mathrm{Re}_2$: Rejection of the hypothesis $H:p\leq p_0$, i.e. p is considered to exceed the tolerance p_0.

4.4 Estimation of the actual proportion nonconforming

The sample proportion nonconforming is

$$\hat{p} = \begin{cases} \frac{x_1}{n_1}, \text{ if the decision procedure terminates in stage1,} \\ \frac{x_1 + x_2}{n_1 + n_2}, \text{ if the decision procedure terminates in stage2.} \end{cases}$$

$\hat{p}$ is an unbiased estimator of the actual proportion nonconforming p in the population. The sampling uncertainty inherent in the estimator p is expressed by a confidence interval. A two-sided confidence interval $D = [p_L; p_U]$ of a nominal level γ satisfies the inequality $P(p_L \le p \le p_U) \ge \gamma$, i.e. with a probability of at least γ, the actual proportion p lies between p_L and p_U. A two-sided confidence interval D for the actual proportion p of misstated items (nonconforming items) can be obtained from the confint function in the ISO 28596 package from the following input quantities: 1) nominal confidence level γ and level of *Trust* chosen for selecting the sampling plan; 2) number x_1 of misstated items (nonconforming items) found in stage 1; 3) if stage 2 was entered: number x_2 of misstated items (nonconforming items) found in stage 2.

5 Application paradigms: lot inspection and financial auditing

Details of two standard application paradigms for the two-stage decision procedure are described below:

— for the inspection of lots of discrete product items, see 6.1

— financial auditing, with two targets: for testing for the compliance of an internal control system (test of controls), and test of details in the course of substantive procedures, see 6.2

5.1 Lot inspection

5.1.1 Sampling

Samples shall be drawn from the lot by simple random sampling. When the lot consists of sub-lots or strata, identified by some rational criterion, representative sampling shall be used in such a way that the number of items sampled is proportional to the number of items in the sub-lot or stratum.

5.1.2 Acceptance of loss

All items in the sample shall be inspected and the nonconforming items shall be counted.

Acceptability of a lot shall be determined by the use of the obtained sampling plans. If the number of nonconforming items found in the sample is equal to or less than the acceptance number Ac_1 and Ac_2, respectively, the lot shall be accepted, otherwise the lot shall not be accepted.

5.1.3 Disposition of non-accepted lots

The disposition of lots not accepted shall be agreed in advance by all interested parties.

5.1.4 Lots with one or more nonconforming units

If a lot has been accepted, the right is reserved not to accept any item found nonconforming during the acceptance sampling inspection that led to lot acceptance.

5.1.5 Resubmitted lots

A lot that has been inspected but not accepted shall only be resubmitted for re-inspection if

a) the purchaser is satisfied that all misstated items (nonconforming items) have been removed or replaced by conforming items, and

b) all interested parties agree.

The responsible authority shall determine the method of re-inspection to be applied.

5.2 Financial auditing

5.2.1 Purposes in the risk-oriented auditing process

The relevant purposes in the risk-oriented auditing process are:

1) test of controls, i.e. tests of compliance in the evaluation of the internal control system (ICS);

2) test of details for selected purposes in course of substantive procedures.

In any case, the auditor notifies the result of the sampling procedure and the subsequent decision in the audit documentation.

5.2.2 Target population, proportion nonconforming and tolerance proportion p_0

In the framework of the evaluation of the ICS, the target population is a totality of internal control events over a specified time frame. The proportion nonconforming is the rate of control events which deviate within a specified time frame from the prescribed internal control procedures. The tolerance proportion p_0 is the rate of deviation from prescribed internal control procedures considered as tolerable for the purposes of financial auditing within a specified time frame.

In the framework of a test of details, the target population is a totality of statements in a specified account balance or class of transactions. The proportion nonconforming is the rate of misstatements in the target population of statements. The tolerance proportion p_0 is the rate of misstatements considered as tolerable for the purposes of financial auditing.

5.2.3 Acceptance and rejection in the case of a test of compliance of the ICS

Both acceptance and rejection affect the auditor’s assessment of the control risk. In the case of acceptance, the auditor rather tends to choose a lower value of the control risk. As a consequence, the amount of auditing efforts in course of subsequent substantive procedures decreases. In the case of rejection, the auditor rather tends to choose a higher value of the control risk. As a consequence, the amount of auditing efforts in course of subsequent substantive procedures increases.

5.2.4 Acceptance and rejection in the case of a test of details

Both acceptance and rejection affect the auditor’s judgment on the existence of material misstatement in the targeted audit population. However, the final conclusion of the auditor is affected by various additional factors, in particular, further test of details, analytical procedures, qualitative assessment of the type of nonconformities.

6 Examples

6.1 Example 1: Lot inspection

A consumer buys a set of screws and can tolerate 3 % of failures. Suppose, the consumer’s confidence in having a low p is mid, i.e. Trust = mid. Furthermore, a nominal confidence level of 0,80 is needed, i.e.

γ = 0,80. Table 2 shall be used, which provides that the sample size in stage 1, n_1 is 63. In addition, using Table 2, the corresponding acceptance and rejection numbers at this stage are $\mathrm{Ac}_1 = 0$ and $\mathrm{Re}_1 = 5$. So, if the inspection leads to no misstated item (nonconforming unit), the lot can be accepted at the first stage. If the inspection reveals 5 or more misstated items (nonconforming units) in the sample of 63, the lot shall be rejected. Otherwise, a second sample shall be drawn. According to Table 2, the sample size in stage 2, n_2 is 228, $\mathrm{Ac}_2 = 8$ and $\mathrm{Re}_2 = 9$.

If the concerned parties are also interested in the operating indicators, Table I.2 provides this additional information: $c.type\ I = 0{,}063\ 0$, $c.type\ II$ = 0,098 8, , $I.ASN$ = 161,67, $I.p_{2\mathrm{nd}} = 0{,}432\ 8$ and $I.cp = 0{,}807\ 8$.

6.2 Example 2: Auditing of an internal control system (purchase process)

An auditor inspects the purchase process of a medium-size retailer of office equipment to evaluate the effectiveness of the respective part of the relevant ICS. In the case subject to auditing, there are a large number of purchases per year with a high quantity of different suppliers. In a first step, the auditor evaluates the appropriateness of the process design. As a result of an interview and observation, the purchase process consists of the following stages: needs assessment, purchase order, incoming goods, invoice receipt and verification, payment processing, adjustment of general ledger. In these stages, numerous different controls have been identified, which shall ensure that the purchase process operates appropriately. After having assessed the appropriateness of the process design, the auditor determines the kind of controls of each stage, which are subject to further investigation. For example, the auditor selects in the stage of "incoming goods" the control, whether the goods delivered correspond to the goods ordered in quantity and quality. Therefore, the auditor prompts the retailer to prove that the responsible staff has duly signed the delivery notes of all incoming goods. The signature should indicate that the quantity and quality of each incoming good have been checked (e. g. information from purchase order) and were considered as appropriate. The auditing target is the proportion p of missing or unsigned or inappropriately signed delivery notes. The auditor assumes 5 % ($p_0 = 0{,}05$) as the tolerable rate of deviation.

In view of the large number of incoming goods, the auditor proceeds by sampling inspection. Calculations in the framework of the risk-oriented auditing process impose for the internal control system (ICS) auditing step a confidence level of $\gamma = 0{,}80$. Concluding from previous experiences with the auditee, the auditor has high confidence in the ICS and assumes the trust level high. Using Table 2, the size of the first sample is n_1=32, with the corresponding acceptance number $\mathrm{Ac}_1 = 0$ and rejection number $\mathrm{Re}_1 = 6$.

The auditor takes a random sample of size n_1=32 of goods incoming events from the ERP system. The inspection of the 32 goods incoming events reveals that all corresponding delivery were duly signed, i.e. the number of nonconforming units in the sample is $x_1 = 0$. Comparing x_1 with the acceptance number $\mathrm{Ac}_1 = 0$, the auditor accepts the hypothesis $H: p \leq p_0$, i.e. the proportion p of missing or unsigned or inappropriately signed delivery notes in the entire population is considered not to exceed the tolerance p_0. Conclusively, the auditor classifies the considered specific internal control procedure as effective.

6.3 Example 3: Auditing of an integral control system (sales process)

An auditor inspects the sales process of a medium-size retailer of steel products to evaluate the effectiveness of the respective part of the relevant internal control system (ICS). There are a large number of sales per year with a high quantity of different customers. In a first step, the auditor evaluates the appropriateness of the sales process design. As a result of an interview and the auditor's own observations, the sales process consists of the following stages: submission of tenders, order acceptance, goods outgoing, invoicing, payment processing, post entries to general ledger. In these stages, numerous different controls have been identified, which shall ensure that the sales process operates appropriately. After having assessed the appropriateness of the process design, the auditor determines the kind of controls of each stage, which are subject to further investigation. For example, the auditor considers in the stage of "invoicing" the control of whether the realisation principle has been observed appropriately. Therefore, the auditor asks the retailer to prove that, with respect to all single sales, the realisation of the turnover has been recorded in the correct period. The auditor would accept 3 % of incorrectly recorded turnovers as tolerable.

In view of the large number of outgoing invoices per year, the auditor proceeds by sampling inspection. For each sampled invoice, the auditor investigates whether the turnover was realised correctly. Calculations in the framework of the risk-oriented auditing process impose for the internal control system (ICS) auditing step a confidence level of $\gamma = 0{,}70$. Concluding from previous experiences with the auditee, the auditor has high confidence in the ICS and assumes the trust level high. Using Table 1, the size of the first sample is n_1=40, with the corresponding acceptance number $\mathrm{Ac}_1 = 0$ and rejection number $\mathrm{Re}_1 = 6$. In the first sample, the auditor observes $x_1 = 7$ incorrectly realized turnovers. Comparing x_1 with the rejection number $\mathrm{Re}_1 = 6$, the auditor rejects the hypothesis $H: p \leq p_0$, i.e. the proportion p of incorrectly realized turnovers in the entire population is considered to exceed the tolerance $p_0 = 0{,}03$, i.e. 5 %. Conclusively, the auditor classifies the considered specific internal control procedure as ineffective.

6.4 Example 4: Auditing test of details (accounts receivable)

An auditor inspects the accounts receivables of a medium-size retailer of office equipment with respect to accuracy of statements at balance sheet date. The auditor imposes a tolerance of $p_0 = 0{,}05$, i.e. 5 % of misstatements at balance sheet date are assumed as tolerable.

In view of the large number of accounts receivable, the auditor proceeds by sampling inspection. Calculations in the framework of the risk-oriented auditing process impose for the test of details auditing step a confidence level of $\gamma = 0{,}70$. Concluding from previous bad experiences with the auditee, the auditor has low confidence in the client's accounting and assumes the trust level low. Using Table 1, the size of the first sample is n_1=36, with the corresponding acceptance number $\mathrm{Ac}_1 = 0$ and rejection number $\mathrm{Re}_1 = 4$. In the first sample, the auditor observes $x_1 = 0$ misstatements. Comparing x_1 with the acceptance number $\mathrm{Ac}_1 = 0$, the auditor accepts the hypothesis $H: p \leq p_0$, i.e. the proportion p of misstated accounts in the entire accounts receivables population is considered not to exceed the tolerance p_0.

6.5 Example 5: Auditing test of details (raw materials)

An auditor inspects the raw materials inventory of a medium-size retailer of steel products with respect to accurate value assessment at balance sheet date. The auditor imposes a tolerance of $p_0 = 0{,}05$, i.e. 5 % of inaccurate value assessments at balance sheet date are assumed as tolerable.

In view of the large variety of raw materials, the auditor proceeds by sampling inspection. The auditor imposes for the test of details a confidence level of $\gamma = 0{,}90$. Concluding from previous experiences with the auditee, the auditor has moderate confidence in the client's materials assessment and assumes the trust level mid. Using Table 3, the size of the first sample is n_1=52, with the corresponding acceptance number $\mathrm{Ac}_1 = 0$ and rejection number $\mathrm{Re}_1 = 7$. In the first sample, the auditor observes $x_1 = 4$ wrong assessments. Conclusively, the auditor proceeds to a second sample of size n_2=185 which is found to contain $x_2 = 7$ wrong assessments. In the second stage, the acceptance number is $\mathrm{Ac}_2 = 11$ and the rejection number is $\mathrm{Re}_2 = 12$. The cumulative number of wrong assessments is $x_1 + x_2 = 11$. Conclusively, the auditor accepts the hypothesis $H: p \leq p_0$, i.e. the proportion p of wrong assessments in the entire population of inventory data is considered not to exceed the tolerance p_0.

7 Sampling plans

Sampling plans with acceptance/rejection numbers for varying nominal confidence levels, stages arranged row-wise are given in Tables 1 to 5.

Table 1 — Sampling plans under nominal confidence level γ = 0,70

Trust	p_0 in proportion					
	0,01	0,02	0,03	0,04	0,05	0,06
Low	181 (0; 4)	91 (0; 4)	60 (0; 4)	45 (0; 4)	36 (0; 4)	30 (0; 4)
	797 (9; 10)	449 (10; 11)	393 (13; 14)	299 (13; 14)	260 (14; 15)	217 (14; 15)
Medium	148 (0; 4)	74 (0; 4)	49 (0; 4)	37 (0; 4)	30 (0; 4)	25 (0; 4)
	599 (7; 8)	299 (7; 8)	200 (7; 8)	150 (7; 8)	120 (7; 8)	100 (7; 8)
High	120 (0; 7)	60 (0; 6)	40 (0; 6)	30 (0; 6)	24 (0; 5)	20 (0; 5)
	557 (6; 7)	278 (6; 7)	147 (5; 6)	126 (6; 7)	103 (5; 6)	82 (5; 6)
Trust	p_0 in proportion					
	0,07	0,08	0,09	0,1	0,15	0,2
Low	26 (0; 4)	22 (0; 4)	20 (0; 4)	18 (0; 4)	12 (0; 4)	9 (0; 4)
	185 (14; 15)	150 (13; 14)	121 (12; 13)	109 (12; 13)	58 (10; 11)	44 (10; 11)
Medium	22 (0; 4)	19 (0; 4)	17 (0; 4)	15 (0; 4)	10 (0; 4)	8 (0; 4)
	85 (7; 8)	75 (7; 8)	66 (7; 8)	50 (6; 7)	41 (7; 8)	29 (7; 8)
High	17 (0; 5)	15 (0; 5)	13 (0; 5)	12 (0; 5)	8 (0; 4)	6 (0; 4)
	64 (5; 6)	59 (5; 6)	56 (5; 6)	38 (4; 5)	34 (5; 6)	23 (5; 6)

Table 2 — Sampling plans under nominal confidence level γ = 0,80

Trust	p_0 in proportion					
	0,02	0,03	0,04	0,05	0,06	0,07
Low	110 (0; 5)	74 (0; 5)	55 (0; 5)	44 (0; 5)	37 (0; 5)	31 (0; 5)
	600 (13; 14)	460 (15; 16)	343 (15; 16)	275 (15; 16)	229 (15; 16)	197 (15; 16)
Medium	94 (0; 5)	63 (0; 5)	47 (0; 5)	38 (0; 5)	32 (0; 5)	27 (0; 5)
	392 (9; 10)	228 (8; 9)	169 (8; 9)	135 (8; 9)	112 (8; 9)	97 (8; 9)
High	80 (0; 7)	53 (0; 8)	40 (0; 7)	32 (0; 6)	27 (0; 6)	23 (0; 6)
	333 (7; 8)	214 (7; 8)	151 (7; 8)	113 (6; 7)	94 (6; 7)	81 (6; 7)
Trust	p_0 in proportion					
	0,08	0,09	0,1	0,15	0,2	
Low	27 (0; 5)	24 (0; 5)	22 (0; 5)	15 (0; 5)	11 (0; 5)	
	160 (14; 15)	142 (14; 15)	128 (14; 15)	76 (13; 14)	48 (11; 12)	
Medium	24 (0; 5)	21 (0; 5)	19 (0; 5)	13 (0; 5)	10 (0; 5)	
	85 (8; 9)	76 (8; 9)	57 (7; 8)	52 (9; 10)	33 (8; 9)	
High	20 (0; 6)	18 (0; 6)	16 (0; 6)	10 (0; 6)	8 (0; 5)	
	71 (6; 7)	64 (6; 7)	52 (6; 7)	37 (6; 7)	26 (6; 7)	

Table 3 — Sampling plans under nominal confidence level γ = 0,90

Trust	**p_0 in proportion**					
	0,02	0,03	0,04	0,05	0,06	0,07
Low	150 (0; 7)	100 (0; 7)	75 (0; 7)	60 (0; 7)	50 (0; 7)	43 (0; 7)
	710 (16; 17)	547 (18; 19)	407 (18; 19)	306 (17; 18)	255 (17; 18)	219 (17; 18)
Medium	131 (0; 7)	88 (0; 7)	66 (0; 7)	52 (0; 7)	44 (0; 7)	38 (0; 7)
	463 (11; 12)	309 (11; 12)	232 (11; 12)	185 (11; 12)	154 (11; 12)	132 (11; 12)
High	114 (0; 11)	76 (0; 9)	57 (0; 8)	45 (0; 9)	38 (0; 8)	32 (0; 8)
	440 (10; 11)	275 (9; 10)	211 (9; 10)	163 (9; 10)	139 (9; 10)	119 (9; 10)
Trust	**p_0 in proportion**					
	0,08	0,09	0,1	0,15	0,2	
Low	37 (0; 7)	33 (0; 7)	30 (0; 7)	17 (0; 6)	13 (0; 6)	
	192 (17; 18)	171 (17; 18)	141 (16; 17)	83 (14; 15)	51 (12; 13)	
Medium	33 (0; 7)	29 (0; 7)	26 (0; 7)	16 (0; 7)	12 (0; 6)	
	116 (11; 12)	92 (10; 11)	72 (9; 10)	62 (11; 12)	42 (10; 11)	
High	28 (0; 8)	25 (0; 7)	22 (0; 8)	15 (0; 7)	11 (0; 6)	
	104 (9; 10)	81 (8; 9)	63 (7; 8)	53 (9; 10)	37 (9; 10)	

Table 4 — Sampling plans under confidence value $\gamma = 0,99$

Trust	p_0 in proportion					
	0,02	0,03	0,04	0,05	0,06	0,07
Low	188 (0; 9)	126 (0; 9)	94 (0; 9)	76 (0; 9)	63 (0; 9)	54 (0; 9)
	802 (18; 19)	631 (21; 22)	472 (21; 22)	357 (20; 21)	298 (20; 21)	240 (19; 20)
Medium	169 (0; 9)	112 (0; 9)	84 (0; 9)	67 (0; 9)	56 (0; 8)	48 (0; 8)
	553 (13; 14)	363 (13; 14)	271 (13; 14)	216 (13; 14)	201 (14; 15)	184 (15; 16)
High	149 (0; 11)	99 (0; 10)	74 (0; 10)	59 (0; 10)	49 (0; 10)	42 (0; 10)
	500 (11; 12)	332 (11; 12)	253 (11; 12)	199 (11; 12)	177 (12; 13)	166 (13; 14)
Trust	p_0 **in proportion**					
	0,08	0,09	0,1	0,15	0,2	
Low	47 (0; 9)	38 (0; 8)	34 (0; 8)	23 (0; 8)	17 (0; 8)	
	208 (19; 20)	189 (19; 20)	171 (19; 20)	99 (17; 18)	64 (15; 16)	
Medium	42 (0; 8)	35 (0; 8)	32 (0; 8)	21 (0; 8)	16 (0; 8)	
	165 (15; 16)	144 (15; 16)	130 (15; 16)	72 (13; 14)	49 (12; 13)	
High	36 (0; 11)	32 (0; 10)	29 (0; 10)	19 (0; 9)	14 (0; 8)	
	148 (13; 14)	132 (13; 14)	117 (13; 14)	58 (10; 11)	44 (10; 11)	

Table 5 — Sampling plans under nominal confidence level $\gamma = 0{,}99$

Trust	p_0 in proportion									
	0,03		0,04		0,05		0,06		0,07	
Low	177	(0; 13)	133	(0; 13)	107	(0; 13)	89	(0; 13)	76	(0; 13)
	806	(27; 28)	603	(27; 28)	481	(27; 28)	402	(27; 28)	343	(27; 28)
Medium	165	(0; 13)	123	(0; 13)	99	(0; 13)	82	(0; 13)	70	(0; 12)
	578	(20; 21)	408	(19; 20)	302	(18; 19)	283	(20; 21)	263	(21; 22)
High	152	(0; 17)	113	(0; 15)	90	(0; 15)	75	(0; 17)	64	(0; 16)
	540	(18; 19)	374	(17; 18)	284	(16; 17)	268	(18; 19)	240	(19; 20)
Trust	p_0 in proportion									
	0,08		0,09		0,1		0,15		0,2	
Low	62	(0; 12)	55	(0; 12)	50	(0; 12)	33	(0; 12)	23	(0; 11)
	305	(27; 28)	259	(26; 27)	222	(25; 26)	126	(22; 23)	91	(21; 22)
Medium	59	(0; 12)	52	(0; 12)	47	(0; 12)	31	(0; 12)	22	(0; 11)
	231	(21; 22)	192	(20; 21)	152	(18; 19)	94	(17; 18)	56	(14; 15)
High	56	(0; 16)	49	(0; 16)	44	(0; 15)	29	(0; 15)	21	(0; 13)
	209	(19; 20)	178	(18; 19)	129	(15; 16)	86	(15; 16)	48	(12; 13)

Annex A
(informative)

Confidence intervals

The methodology of the decision procedure in 4.3 is based on shortest two-sided confidence intervals under prior information for an unknown proportion. See Reference [5] for technical details on shortest confidence intervals under prior information. The hypothesis test compares the confidence intervals against the desired tolerance p_0.

The decision procedure in stages 1 and 2 proceeds as follows, see Figure A.1 for an illustration.

In stage 1, after drawing n_1 items and observing x_1 nonconforming units, a confidence interval for p is calculated. The decision in stage 1 proceeds according to the following scenarios:

1.1 When the confidence interval lies completely below the tolerance, the lot is accepted.

1.2 When the confidence interval completely lies above the tolerance, the lot is rejected.

1.3 If the tolerance lies within the confidence interval, no decision can be made in stage 1 and the second sampling step (stage 2) is necessary.

In stage 2, a number x_2 of nonconforming items is observed in a sample of size n_2. A new confidence interval for p is calculated based on the total number $x_1 + x_2$ of nonconforming units. The decision in stage 2 proceeds according to the following scenarios:

2.1 When the confidence interval is completely above or below the tolerance, the decision is approval and rejection, is analogous to the first stage.

2.2 If the interval contains the tolerance, the decision is based on whether a bigger portion of the confidence interval is below or above the tolerance, to result in acceptance or rejection.

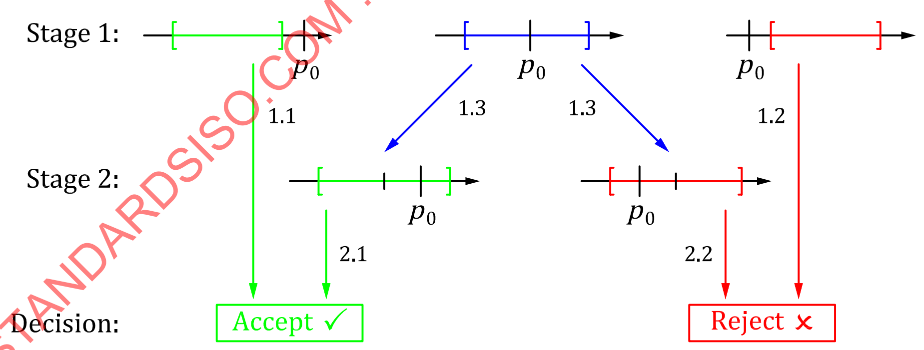

Figure A.1 — Illustration of two step decision procedure based on position of confidence intervals

The relationship between confidence intervals and the decision procedure is illustrated in Figures A.2 and A.3 The blue horizontal line segments represent the confidence intervals for p relative to the sample observation x on the vertical axis. For the first stage, the confidence intervals are intersected with the tolerance p_0=0,3. Figure A.2 shows the largest x_1 where p_0 is above or equal to the upper confidence limit and the smallest x where p_0 is below the lower confidence limit. This results in the acceptance number Ac_1 =3 and rejection number Re_1 =12. For the second stage, the highest point lower or equal to p_0 determines Ac_2 =7 and Re_2 =8, see Figure A.3.

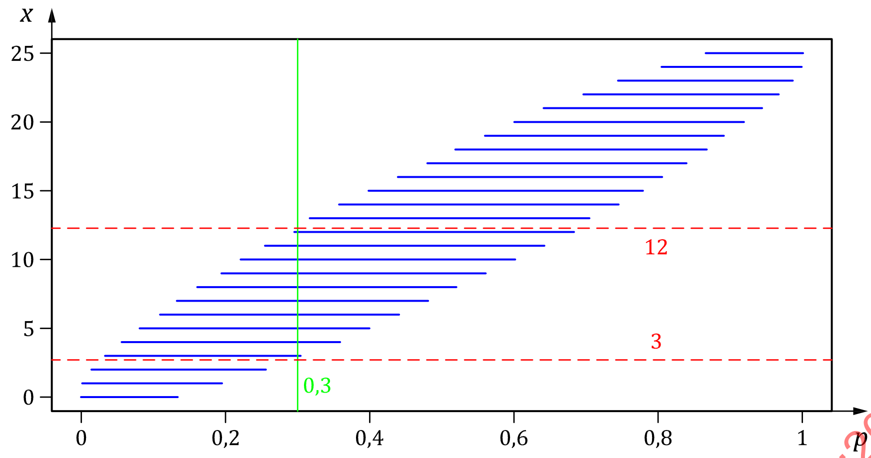

Figure A.2 — Confidence intervals for p (horizontal axis) indexed in the number x_1 (vertical axis) of misstatements (nonconforming items) for sample size n_1 = 25

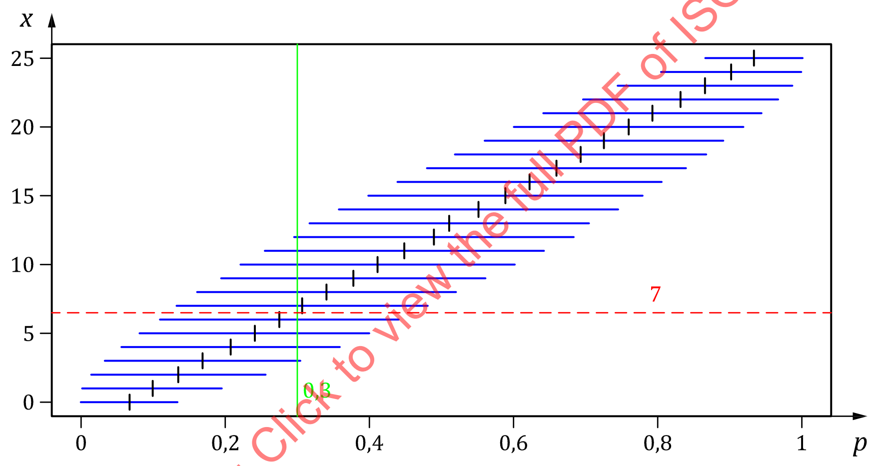

Figure A.3 — Confidence intervals for p (horizontal axis) with marked middle points, indexed in the number x_1+ x_2 (vertical axis) of misstatements (nonconforming items) for sample size n_1+ n_2 = 25

Due to the two-stage procedure, the actual confidence level of the final confidence interval undercuts the prescribed nominal confidence level γ in a small interval around p_0. See Figures G.1 to G.5 in Annex G.

Annex B
(informative)

Operating characteristics (OC)

The operating characteristic (OC) denotes the function of acceptance of a statistical test procedure. In the present case, for a set of parameters $n_1, (\mathrm{Ac}_1; \mathrm{Re}_1), n_2, \mathrm{Ac}_2$ and $X_1 \sim Bi(n_1, p), X_2 \sim Bi(n_2, p)$ the operating characteristic function can be formulated as

$$P_a^{(2)}(p) = P(accept\ H_0 | p) = P(X_1 \leq or \{\mathrm{Ac}_1 < X_1 < \mathrm{Re}_1 and\ X_1 + X_2 \leq \mathrm{Ac}_2\} | p)$$

$$= \sum_{x_1=0}^{\mathrm{Ac}_1} P(X_1 = x_1 | p) + \sum_{x_1=\mathrm{Ac}_1+1}^{\mathrm{Re}_1-1} P(X_1 = x_1 | p) \sum_{x_2=0}^{\mathrm{Ac}_2 - x_1} P(X_2 = x_2 | p)$$

$$= \sum_{x_1=0}^{\mathrm{Ac}_1} \binom{n_1}{x_1} p^{x_1} (1-p)^{n_1 - x_1} + \sum_{x_1=\mathrm{Ac}_1+1}^{\mathrm{Re}_1-1} \sum_{x_2=0}^{\mathrm{Ac}_2 - x_1} \binom{n_1}{x_1} \binom{n_2}{x_2} p^{x_1 + x_2} (1-p)^{n_1 + n_2 - x_1 - x_2}$$

Annex C
(informative)

OC matching

The OC function described in Annex B can be used to denote two different sampling plans as equally “fair”, if their OC functions are nearly equal. The so-called OC matching principle uses this requirement of fairness to obtain a single stage sampling plan that matches a given two-stage sampling plan.

Given (n, Ac) and $X \sim Bi(n,p)$, the one stage OC is denoted by:

$$P_a^{(1)}(p) = P(accept\ H_0|p) = \sum_{x=0}^{\mathrm{Ac}} \binom{n}{x} p^x (1-p)^{n-x}$$

which is the binomial OC. To find the OC matched sample size n_{match} to a given plan n_1, n_2 one has to minimize the squared residual:

$$\min_{n_1 < n \leq n_1 + n_2} R^2(n, n_1, n_2) := \min_{n_1 < n \leq n_1 + n_2} \int_0^1 \left(P_a^{(1)}(p) - P_a^{(2)}(p)\right)^2 \mathrm{d}p$$

Values smaller or equal to n_1 are not considered as valid for n, since they make the calculated two-stage sampling plan completely obsolete. Larger values than $n_1 + n_2$ are also not to be considered, as they render the one stage sampling plan worthless.

Annex D
(informative)

Conditional type I and II errors (conditional risks)

Given $n_1, (\mathrm{Ac}_1; \mathrm{Re}_1), n_2, \mathrm{Ac}_2, a, b, X_1 \sim Bi(n_1, y), X_2 \sim Bi(n_2, y)$ and $p = y$ with $Y \sim Beta(a, b)$, the conditional type I and type II errors can be derived as:

$$P(\text{erroneous acceptance})$$

$$= \int_{p_0}^{1} \left(\sum_{x_1=0}^{\mathrm{Ac}_1} P(X_1 = x_1 | Y = y) + \sum_{x_1=\mathrm{Ac}+1}^{\mathrm{Re}_1-1} P(X_1 = x_1 | Y = y) \sum_{x_2=0}^{\mathrm{Ac}_2-x_1} P(X_2 = x_2 | Y = y) \right) f_Y(y) \mathrm{d}y$$

$$= \sum_{x_1=0}^{\mathrm{Ac}_1} \frac{\binom{n_1}{x_1}}{B(a,b)} \left(B(x_1 + a, n_1 - x_1 + b) - B(p_0; x_1 + a, n_1 - x_1 + b) \right)$$

$$+ \sum_{x_1=\mathrm{Ac}_1+1}^{\mathrm{Re}_1-1} \sum_{x_2=0}^{\mathrm{Ac}_2=x_1} \frac{\binom{n_1}{x_1}\binom{n_2}{x_2}}{B(a,b)} \left(B(x_1 + x_2 + a, n_1 + n_2 - x_1 - x_2 + b) \right)$$

$$-B(p_0; x_1 + x_2 + a, n_1 + n_2 - x_1 - x_2 + b)$$

$$c.type\ I := \frac{P(\text{erroneous acceptance})}{P(p > p_0)} = \frac{P(\text{erroneous acceptance})}{\frac{B(a,b) - B(p_0; a,b)}{B(a,b)}}$$

$$= \sum_{x_1=0}^{\mathrm{Ac}_1} \frac{\binom{n_1}{x_1} \left(B(x_1 + a, n_1 - x_1 + b) - B(p_0; x_1 + a, n_1 - x_1 + b) \right)}{B(a,b) - B(p_0; a,b)}$$

$$+ \sum_{x_1=\mathrm{Ac}_1+1}^{\mathrm{Re}_1-1} \sum_{x_2=0}^{\mathrm{Ac}_2-x_1} \binom{n_1}{x_1}\binom{n_2}{x_2} \left(\frac{B(x_1 + x_2 + a, n_1 + n_2 - x_1 - x_2 + b)}{B(a,b) - B(p_0; a,b)} \right.$$

$$\left. - \frac{B(p_0; x_1 + x_2 + a, n_1 + n_2 - x_1 - x_2 + b)}{B(a,b) - B(p_0; a,b)} \right)$$

and

$$P(\text{erroneous rejection})$$

$$= \int_{0}^{p_0} \left(\sum_{x_1=\mathrm{Re}_1}^{n_1} P(X_1 = x_1 | Y = y) + \sum_{x_1=\mathrm{Ac}_1+1}^{\mathrm{Re}_1-1} P(X_1 = x_1 | Y = y) \sum_{x_2=\mathrm{Ac}_2-x_1+1}^{n_2} P(X_2 = x_2 | Y = y) \right) f_Y(y) \mathrm{d}y$$

$$= \sum_{x_1=\mathrm{Re}_1}^{n_1} \frac{\binom{n_1}{x_1}}{B(a,b)} B(p_0; x_1 + a, n_1 - x_1 + b) + \sum_{x_1=\mathrm{Ac}_1+1}^{\mathrm{Re}_1-1} \sum_{x_2=\mathrm{Ac}_2}^{n_2} \frac{\binom{n_1}{x_1}\binom{n_2}{x_2}}{B(a,b)} B(p_0; x_1 + x_2 + a, n_1 + n_2 - x_1 - x_2 + b)$$

$$c.type\ II := \frac{P(\text{erroneous rejection})}{P(p \leq p_0)} = \frac{P(\text{erroneous rejection})}{\dfrac{B(p_0;a,b)}{B(a,b)}}$$

$$= \sum_{x_1=\mathrm{Re}_1}^{n_1} \frac{\binom{n_1}{x_1} B(p_0;x_1+a,n_1-x_1+b)}{B(p_0;a,b)}$$

$$+ \sum_{x_1=\mathrm{Ac}_1+1}^{\mathrm{Re}_1-1} \sum_{x_2=\mathrm{Ac}_2}^{n_2} \frac{\binom{n_1}{x_1}\binom{n_2}{x_2} B(p_0;x_1+x_2+a,n_1+n_2-x_1-x_2+b)}{B(p_0;a,b)}$$

Annex E
(informative)

Integrated second stage probability

If the integrated probability of entering the second stage, $I.p_{2nd}$, gets near to 1, first drawing and inspecting a sample size n_1 becomes obsolete, since nearly every time drawing $n_1 + n_2$ is needed.

$I.p_{2nd}$ decreases with increasing n_1.

Given $n_1, a, b, p_0, X_1 \sim Bi(n_1, y)$ and therefore also given $(\mathrm{Ac}_1; \mathrm{Re}_1)$, this probability can be derived as:

$$I.p_{2nd} := P(\text{requiring second step})$$

$$= \int_0^1 P(\text{requring second step} | Y = y) f_Y(y) \mathrm{d}y$$

$$= \int_0^1 P(\mathrm{Ac}_1 < x_1 < \mathrm{Re}_1 | Y = y) f_Y(y) \mathrm{d}y$$

$$= \int_0^1 \left(\sum_{x_1 = \mathrm{Ac}_1 + 1}^{\mathrm{Re}_1 - 1} P(X_1 = x_1 | Y = y) \right) f_Y(y) \mathrm{d}y$$

$$= \sum_{x_1 = \mathrm{Ac}_1 + 1}^{\mathrm{Re}_1 - 1} \frac{\binom{n_1}{x_1}}{B(a,b)} \int_0^1 y^{x_1 + a - 1} (1 - y)^{n_1 - x_1 + b - 1} \mathrm{d}y$$

$$= \sum_{x_1 = \mathrm{Ac}_1 + 1}^{\mathrm{Re}_1 - 1} \frac{\binom{n_1}{x_1}}{B(a,b)} B(x_1 + a, n_1 - x_1 + b)$$

Annex F
(informative)

Integrated average sample number

The integrated average sample number, $I.ASN(n_1, n_2)$, can be derived using the integrated probability for entering second stage, $I.p_{2nd}$, from Annex E, above, since

$$I.ASN(n_1, n_2) = \int_0^1 (n_1 + n_2 \times P(\text{requiring second step} | Y = y))\, f_Y(y)\,\mathrm{d}y$$

$$= \int_0^1 n_1 f_Y(y)\,\mathrm{d}y + n_2 \int_0^1 P(\text{requiring second step} | Y = y)\, f_Y(y)\,\mathrm{d}y$$

$$= n_1 + n_2 \times I.p_{2nd}$$

Annex G
(informative)

Actual coverage probability

Let $D_{x_1}^{n_1}$ be the confidence interval calculated from the realization x_1 in the first sample n_1 and $D_{x_1,x_2}^{n_1,n_2}$ the confidence interval calculated from realization x_1 in the first and x_2 in the second sample n_2. Then, the actual coverage probability cp for given $p, n_1, (\mathrm{Ac}_1; \mathrm{Re}_1), n_2, \mathrm{Ac}_2$ is obtained as:

$$cp = P\left(p \in D^{n_1,n_2}\right)$$

$$= \sum_{x_1=0, x_1 \notin (\mathrm{Ac}_1+1,\ldots,\mathrm{Re}_1-1)}^{n_1} P\left(p \in D_{x_1}^{n_1}\right) P(X_1 = x_1 | p)$$
$$+ \sum_{x_1=\mathrm{Ac}_1+1}^{\mathrm{Re}_1-1} P(X_1 = x_1 | p) \sum_{x_2=0}^{n_2} P\left(p \in D_{x_1,x_2}^{n_1,n_2}\right) P(X_2 = x_2 | p)$$
$$= \sum_{x_1=0, x_1 \notin (\mathrm{Ac}_1+1,\ldots,\mathrm{Re}_1-1)}^{n_1} \|_{p \in D_{x_1}^{n_1}} P(X_1 = x_1 | p)$$
$$+ \sum_{x_1=\mathrm{Ac}_1+1}^{\mathrm{Re}_1-1} P(X_1 = x_1 | p) \sum_{x_2=0}^{n_2} \|_{p \in D_{x_1,x_2}^{n_1,n_2}} P(X_2 = x_2 | p)$$

and the integrated coverage probability is obtained as:

$$I.cp = \int_0^1 cp f_Y(y) \mathrm{d}y$$

where $\|$ is the indicator function, defined by:

$$\|_{p \in D} := \begin{cases} 1 \; if \; p \in D \\ 0 \; if \; p \notin D \end{cases}.$$

Some illustrative graphics of the coverage probability, as well as their generating software function, are given in Figures G.1 to G.5.

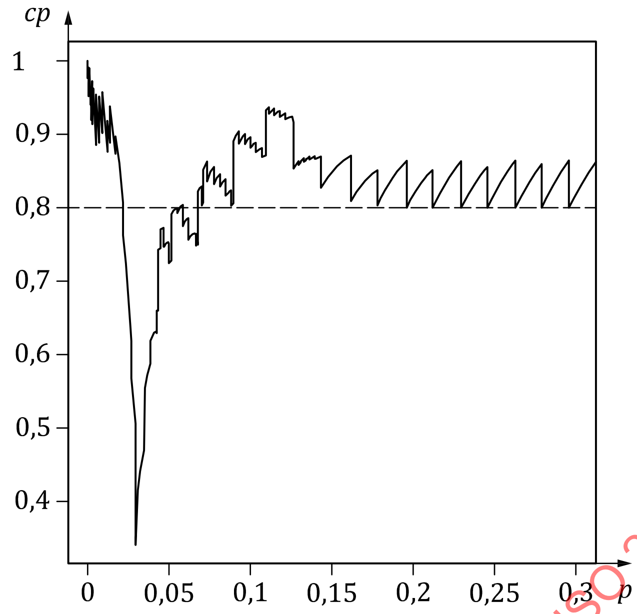

Figure G.1 — Actual coverage probability for Example 1
(Given Trust level *mid*, nominal confidence level γ = 0,80 and tolerance $p_0 = 0{,}03$, the graphic is generated by coverage(Trust = “Mid”, gamma = 0,80, p0 = 0,03, p_capped = TRUE))

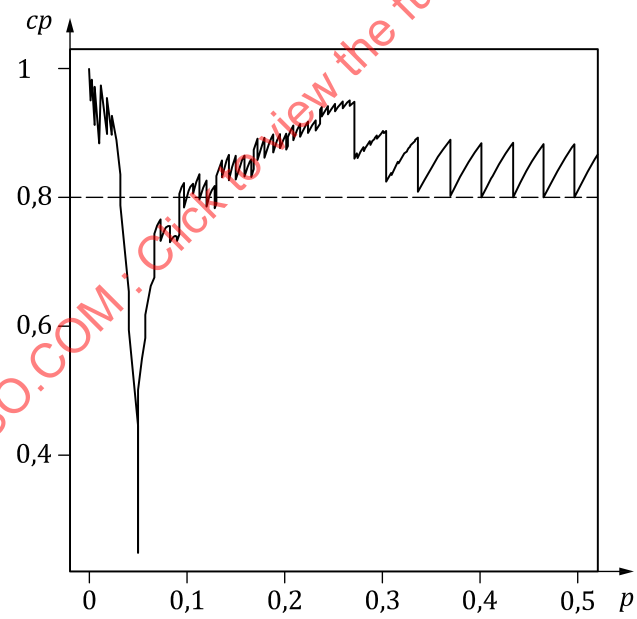

Figure G.2 — Actual coverage probability for Example 2
(Given Trust level *high*, nominal confidence level γ = 0,80 and tolerance $p_0 = 0{,}05$, the graphic is generated by coverage(Trust = “High”, gamma = 0,80, p0 = 0,05, p_capped = TRUE))

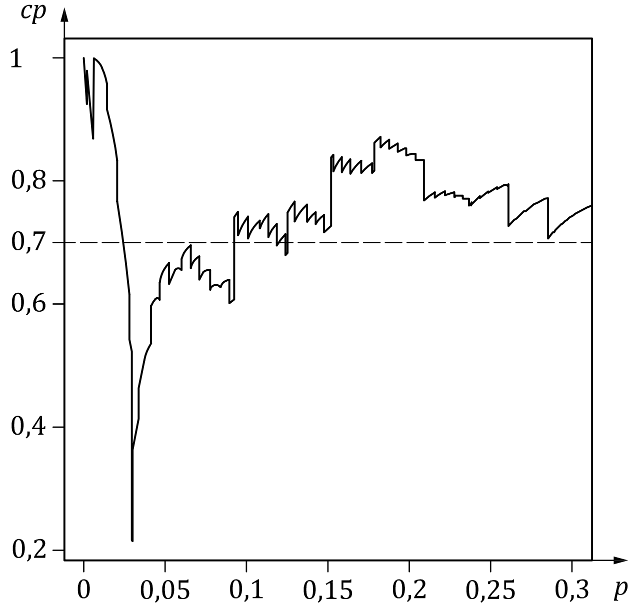

Figure G.3 — Actual coverage probability for Example 3
(Given Trust level *high*, nominal confidence level γ = 0,70 and tolerance $p_0 = 0{,}03$, the graphic is generated by coverage(Trust = "High", gamma = 0,70, p0 = 0,03, p_capped = TRUE))

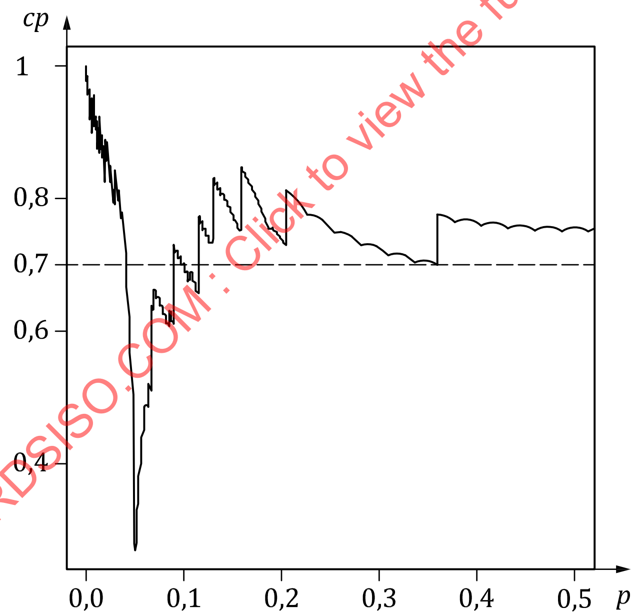

Figure G.4 — Actual coverage probability for Example 4
(Given Trust level *low*, nominal confidence level γ = 0,70 and tolerance $p_0 = 0{,}05$, the graphic is generated by coverage(Trust = "Low", gamma = 0,70, p0 = 0,05, p_capped = TRUE))

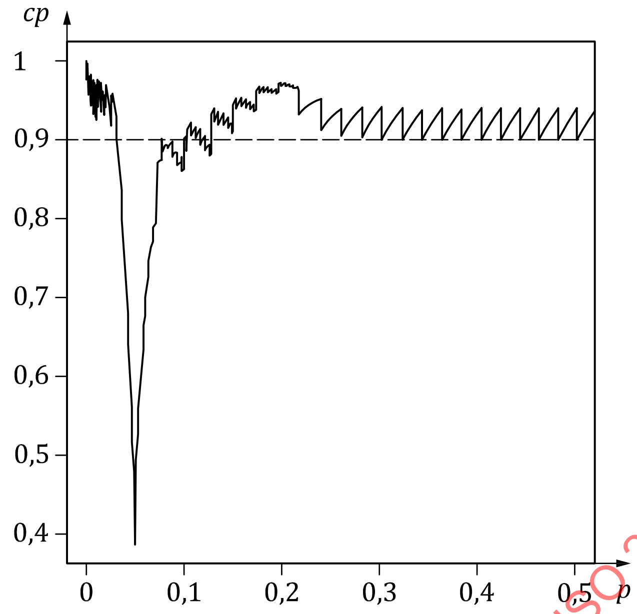

Figure G.5 — Actual coverage probability for Example 5
(Given Trust level *mid*, nominal confidence level γ = 0,90 and tolerance $p_0 = 0{,}05$, the graphic is generated by coverage(Trust = “Mid”, gamma = 0,90, p0 = 0,05, p_capped = TRUE))

Annex H
(informative)

Prior information model

The prior knowledge level used to obtain the appropriate sampling plan from Table 1 to Table 5 is represented by two parameters *a* and *b*. These two parameters uniquely define a beta distribution that models the prior knowledge given on the proportion of misstated items (nonconforming items), i.e. how probable are low values of *p*, how probable are high values, etc. Depending on the choice of *a* and *b*, two shapes have to be distinguished.

a) Choosing $a=b=1$ results in a uniform distribution of *p*, i.e. all values of *p* are considered equally likely. This case expresses the absence of specific prior knowledge, and corresponds to the trust level *low* in the sampling plan tables. See Figure H.1.

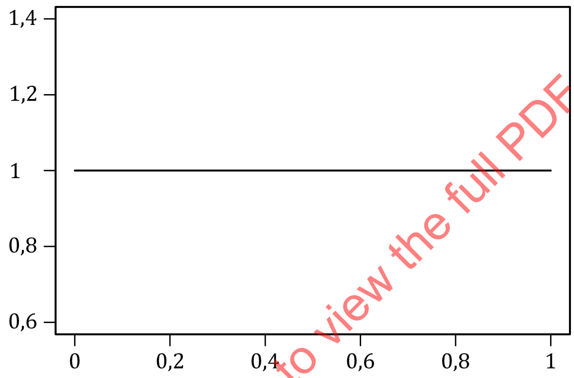

Figure H.1 — Density of the uniform Beta distribution, resembling low Trust level

b) Choosing $a \leq 1$ and $b \geq 1$ (but not $a=b=1$) results in a strictly decreasing beta curve, which can be used for representing mid and high Trust levels. This means that low values of *p* are more likely to occur than high values. Trust level high represents the best possible statistical prior information, resulting in the best possible sample size reduction. Trust level mid is chosen so that the first sample size is the geometric mean between the first sample sizes of high and low Trust levels, rounded up. See Figure H.2.

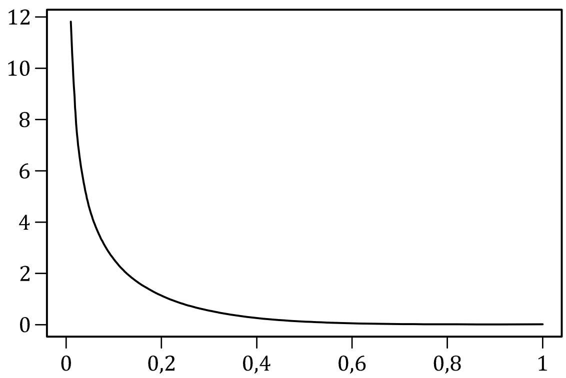

Figure H.2 — Density of the strictly decreasing Beta distribution; example for mid or high Trust levels

A quality manager who intends to inspect a lot may choose the Trust level based on past records of the lot inspection process. An auditor may choose the Trust level by considering past experiences with the auditee and observation from previous auditing steps.

Annex I
(informative)

Operating indicators of sampling plans

Table 1 to Table 5 are constructed for operating the decision procedure of this document. The subsequent Table I.1 through to Table I.5 display statistical operating indicators of the sampling plans provided by Table 1 to Table 5.

The cell entries in the subsequent Table I.1 through Table I.5 are explained by Figure I.1. See Clause 3 for the symbols and abbreviated terms.

p_0 in proportion	
c.type I conditional probability of erroneous acceptance	*c.type II* conditional probability of erroneous rejection
I.ASN integrated average sample number	$I.p_{2nd}$ integrated probability of entering the second stage
n_{match} one stage sample size with same OC as two stage sampling plan	*I.cp* integrated actual coverage
a first shape parameter of the beta distribution	*b* second shape parameter of the beta distribution

Figure I.1 — Cell entries in Table I.1 through Table I.5

Table I.1 — Operating indicators for plans in Table 1 under nominal confidence level γ = 0,70

Trust	p_0 in proportion											
	0,01		0,02		0,03		0,04		0,05		0,06	
low	0,001 9	0,114 5	0,003 7	0,111 2	0,005 4	0,096 1	0,006 9	0,099 7	0,008 4	0,097 9	0,010 0	0,098 0
	194,14	0,016 5	105,64	0,032 6	79,33	0,049 2	64,50	0,065 2	57,08	0,081 1	51,00	0,096 8
	754,00	0,722 1	427,00	0,729 1	313,00	0,737 2	237,00	0,737 4	210,00	0,749 7	175,00	0,749 7
	1	1	1	1	1	1	1	1	1	1	1	1
mid	0,083 8	0,091 2	0,086 0	0,090 7	0,086 3	0,090 8	0,085 4	0,091 5	0,081 8	0,093 2	0,083 8	0,093 0
	378,19	0,384 3	191,37	0,392 6	128,45	0,397 3	96,94	0,399 6	77,50	0,395 9	65,44	0,404 4
	520,00	0,738 4	260,00	0,740 0	174,00	0,740 4	131,00	0,738 2	105,00	0,740 8	88,00	0,742 7
	1	35	1	18	1	12	1	9	1	7	1	6
high	0,099 8	0,003 9	0,099 7	0,012 7	0,099 6	0,012 1	0,099 4	0,008 4	0,100 0	0,049 4	0,099 0	0,043 7
	153,67	0,060 4	105,66	0,164 2	62,58	0,153 6	47,91	0,142 2	58,23	0,332 3	46,48	0,322 9
	418,00	0,972 4	209,00	0,923 0	109,00	0,924 5	100,00	0,930 0	71,00	0,837 4	58,00	0,842 5
	0,03	8	0,1	5	0,09	3	0,08	2	0,31	4	0,29	3
Trust	p_0 **in proportion**											
	0,07		0,08		0,09		0,1		0,15		0,2	
low	0,011 5	0,097 3	0,014 0	0,097 1	0,015 8	0,099 2	0,017 5	0,099 2	0,028 7	0,098 0	0,036 5	0,099 4
	46,56	0,111 1	41,57	0,130 4	37,29	0,142 9	35,21	0,157 9	25,38	0,230 8	22,20	0,300 0
	150,00	0,757 8	118,00	0,745 6	105,00	0,754 9	95,00	0,763 5	56,00	0,765 5	42,00	0,768 0
	1	1	1	1	1	1	1	1	1	1	1	1
mid	0,080 0	0,094 2	0,074 2	0,095 8	0,082 9	0,093 1	0,095 1	0,097 6	0,095 9	0,096 3	0,089 8	0,091 7
	55,83	0,398 0	48,13	0,388 4	44,18	0,411 9	36,87	0,437 3	29,50	0,475 5	21,53	0,466 7
	88,00	0,742 0	66,00	0,739 2	69,00	0,740 7	53,00	0,741 7	42,00	0,749 9	31,00	0,753 2
	1	5	1	4	1	4	1	4	1	3	1	2
high	0,098 5	0,032 7	0,099 5	0,032 0	0,098 6	0,039 5	0,073 6	0,037 6	0,099 4	0,008 4	0,100 0	0,049 4
	36,08	0,298 2	31,45	0,278 7	30,07	0,304 8	22,33	0,272 0	47,91	0,142 2	58,23	0,332 3
	47,00	0,848 3	43,00	0,865 2	39,00	0,857 6	35,00	0,856 7	100,00	0,930 0	71,00	0,837 4
	0,25	2	0,22	2	0,25	2	0,21	1	0,08	2	0,31	4

Table I.2 — Operating indicators for plans in Table 2 under nominal level γ = 0,80

Trust	p_0 in proportion											
	0,02		0,03		0,04		0,05		0,06		0,07	
low	0,002 6	0,109 1	0,003 9	0,098 5	0,005 3	0,095 5	0,006 6	0,095 9	0,007 8	0,096 3	0,009 2	0,095 1
	131,62	0,036 0	98,53	0,053 3	79,50	0,071 4	68,44	0,088 9	61,11	0,105 3	55,63	0,125 0
	594,00	0,821 6	425,00	0,823 7	317,00	0,828 0	274,00	0,829 8	229,00	0,827 1	196,00	0,833 2
	1	1	1	1	1	1	1	1	1	1	1	1
mid	0,058 4	0,094 1	0,063 0	0,098 8	0,065 1	0,095 7	0,062 8	0,096 9	0,064 1	0,096 6	0,062 4	0,098 1
	258,08	0,418 6	161,67	0,432 8	120,97	0,437 7	96,54	0,433 6	81,38	0,440 9	69,96	0,442 9
	420,00	0,806 5	249,00	0,807 8	185,00	0,808 1	148,00	0,809 4	124,00	0,810 1	106,00	0,809 5
	1	17	1	12	1	9	1	7	1	6	1	5
high	0,099 7	0,029 4	0,099 7	0,006 7	0,099 0	0,017 4	0,099 6	0,065 5	0,099 3	0,068 2	0,099 1	0,067 5
	176,66	0,290 3	72,65	0,091 8	75,39	0,234 4	82,65	0,448 2	70,18	0,459 4	60,05	0,457 4
	285,00	0,912 3	185,00	0,973 2	157,00	0,928 9	115,00	0,855 0	96,00	0,853 0	83,00	0,852 5
	0,22	15	0,05	7	0,15	5	0,48	8	0,5	7	0,49	6
Trust	p_0 **in proportion**											
	0,08		0,09		0,1		0,15		0,2			
low	0,010 8	0,097 4	0,012 2	0,096 7	0,013 0	0,098 7	0,021 5	0,091 8	0,029 3	0,100 5		
	49,86	0,142 9	46,72	0,160 0	44,26	0,173 9	34,00	0,250 0	27,00	0,333 3		
	159,00	0,832 0	141,00	0,837 0	128,00	0,832 1	77,00	0,831 4	53,00	0,834 8		
	1	1	1	1	1	1	1	1	1	1		
mid	0,056 2	0,101 2	0,062 9	0,099 8	0,075 0	0,099 4	0,071 8	0,093 8	0,070 3	0,096 4		
	60,10	0,424 7	55,73	0,457 0	46,39	0,480 5	39,93	0,517 9	27,00	0,515 2		
	94,00	0,811 5	83,00	0,810 3	64,00	0,807 6	57,00	0,810 4	42,00	0,815 9		
	1	4	1	4	1	4	1	3	1	2		
high	0,098 5	0,061 7	0,097 8	0,070 8	0,099 3	0,045 5	0,098 6	0,034 2	0,096 8	0,077 5		
	50,95	0,435 9	47,78	0,465 2	37,03	0,404 5	22,02	0,324 9	21,56	0,521 5		
	72,00	0,859 8	65,00	0,852 4	55,00	0,874 0	37,00	0,904 2	28,00	0,827 7		
	0,44	5	0,5	5	0,36	3	0,24	2	0,65	2		