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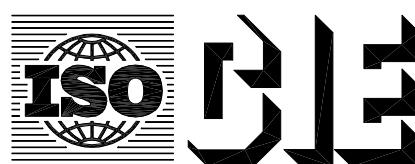
**Colorimetry —**

**Part 3:  
CIE tristimulus values**

*Colorimétrie —*

*Partie 3: Composantes trichromatiques CIE*

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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

ISO 11664-3 was prepared as Standard CIE S 014-3/E by the International Commission on Illumination, which has been recognized by the ISO Council as an international standardizing body. It was adopted by ISO under a special procedure which requires approval by at least 75 % of the member bodies casting a vote, and is published as a joint ISO/CIE edition.

The International Commission on Illumination (abbreviated as CIE from its French title) is an organization devoted to international cooperation and exchange of information among its member countries on all matters relating to the science and art of lighting.

ISO 11664-3 was prepared by CIE Technical Committee 1-57 of Division: *Vision and colour*.

ISO 11664 consists of the following parts, under the general title *Colorimetry*:

- *Part 1: CIE standard colorimetric observers*
- *Part 2: CIE standard illuminants*
- *Part 3: CIE tristimulus values*
- *Part 4: CIE 1976 L\*a\*b\* Colour space*
- *Part 5: CIE 1976 L\*u\*v\* Colour space and u', v' uniform chromaticity scale diagram*

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CIE S 014-3/E:2011

## Colorimetry - Part 3: CIE Tristimulus Values

Colorimétrie - Partie 3: Composantes trichromatiques CIE

Farbmessung - Teil 3: CIE-Farbwerthe

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## Foreword

Standards produced by the Commission Internationale de l'Eclairage are concise documents on aspects of light and lighting that require a unique definition. They are a primary source of internationally accepted and agreed data which can be taken, essentially unaltered, into universal standard systems.

This CIE Standard has been prepared by Technical Committee TC 1-57\* of Division 1 "Vision and Colour" of the Commission Internationale de l'Eclairage and approved by the National Committees of the CIE.

The following ISO and IEC committees and working groups co-operated in the preparation of this standard:

- IEC TC100/TA2 (Audio, video and multimedia systems)
- ISO TC6 (Paper, board and pulps)
- ISO TC35/SC9/WG22 (Paints and varnishes)
- ISO TC38/SC1/WG7 (Textiles)
- ISO TC42 (Photography)
- ISO TC130 (Graphic technology)
- ISO/IEC/JTC1/SC28 (Office systems)

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\* The chairperson of this TC was A.R. Robertson (CA), members were: P.J. Alessi (US), J. Campos Acosta (ES), R. Connelly (US), J.-F. Decarreau (FR), R. Harold (US), R. Hirschler (HU), B. Jordan (CA), C. Kim (KR), D. McDowell (US), P. McGinley (AU), Y. Ohno (US), M.R. Pointer (GB), K. Richter (DE), G. Rösler (DE), J.D. Schanda (HU), R. Sève (FR), K. Witt (DE), H. Yaguchi (JP), J. Zwinkels (CA).

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## Colorimetry - Part 3: CIE Tristimulus Values

### Introduction

Colour stimuli with different spectral distributions can look alike. An important function of colorimetry is to determine which stimuli look alike to a given observer with a given set of colour-matching functions. This is done by calculating a set of three tristimulus values for each stimulus. Equality of tristimulus values indicates equality of colour appearance under equal irradiation and viewing conditions. This Standard is based on long-standing CIE recommendations (CIE, 2004) for the calculation of tristimulus values.

### 1 Scope

This CIE Standard specifies methods of calculating the tristimulus values of colour stimuli for which the spectral distributions are provided. These colour stimuli may be produced by self-luminous light sources or by reflecting or transmitting objects.

The Standard requires that the colour stimulus function be tabulated at measurement intervals of 5 nm or less in a wavelength range of at least 380 nm to 780 nm. Extrapolation methods are suggested for cases where the measured wavelength range is less than 380 nm to 780 nm.

The standard method is defined as summation at 1 nm intervals over the wavelength range from 360 nm to 830 nm. Alternative abridged methods are defined for larger intervals (up to 5 nm) and shorter ranges (down to 380 nm to 780 nm). The alternative methods are to be used only when appropriate and when the user has reviewed the impact on the final results.

The Standard may be used in conjunction with the CIE 1931 standard colorimetric observer or the CIE 1964 standard colorimetric observer.

### 2 Normative References

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

CIE DS 017.2/E:2009. *ILV: International Lighting Vocabulary*.

ISO 11664-1:2007(E)/CIE S 014-1/E:2006. Joint ISO/CIE Standard: *Colorimetry Part 1. CIE Standard Colorimetric Observers*.

ISO 11664-2:2007(E)/CIE S 014-2/E:2006. Joint ISO/CIE Standard: *Colorimetry Part 2. CIE Standard Illuminants*.

ISO 23539:2005(E)/CIE S 010/E:2004. Joint ISO/CIE Standard: *Photometry - The CIE System of Physical Photometry*.

### 3 Definitions, Symbols and Abbreviations

For the purposes of this International Standard, the terms and definitions given in CIE DS 017.2/E:2009 (International Lighting Vocabulary), and the following symbols and abbreviations apply.

$k, k_{10}$	normalizing constants
$K_m$	maximum spectral luminous efficacy of radiation in the CIE standard system of physical photometry

$K_{m,10}$	maximum spectral luminous efficacy of radiation when the $V_{10}(\lambda)$ function is used for photometry
$R(\lambda)$	spectral reflectance factor
$S(\lambda)$	relative spectral distribution of an illuminant
$V(\lambda)$	spectral luminous efficiency function in the CIE standard system of physical photometry
$V_{10}(\lambda)$	spectral luminous efficiency function when the $\bar{y}_{10}(\lambda)$ function is used for photometry
$W_x(\lambda), W_y(\lambda), W_z(\lambda)$	pre-calculated weighting functions for tristimulus integration using the CIE 1931 standard colorimetric observer
$W_{x,10}(\lambda), W_{y,10}(\lambda), W_{z,10}(\lambda)$	pre-calculated weighting functions for tristimulus integration using the CIE 1964 standard colorimetric observer
$x, y, z$	chromaticity coordinates calculated using the CIE 1931 standard colorimetric observer
$x_{10}, y_{10}, z_{10}$	chromaticity coordinates calculated using the CIE 1964 standard colorimetric observer
$\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$	colour-matching functions of the CIE 1931 standard colorimetric observer (also known as the CIE 2° standard colorimetric observer)
$\bar{x}_{10}(\lambda), \bar{y}_{10}(\lambda), \bar{z}_{10}(\lambda)$	colour-matching functions of the CIE 1964 standard colorimetric observer (also known as the CIE 10° standard colorimetric observer)
$X, Y, Z$	tristimulus values calculated using the CIE 1931 standard colorimetric observer
$X_{10}, Y_{10}, Z_{10}$	tristimulus values calculated using the CIE 1964 standard colorimetric observer
$\beta(\lambda)$	spectral radiance factor
$\Delta\lambda$	wavelength interval
$\varphi_i(\lambda)$	colour stimulus function (description of a colour stimulus by the spectral concentration of a radiometric quantity, such as radiance or radiant power, as a function of wavelength)
$\varphi(\lambda)$	relative colour stimulus function (relative spectral distribution of the colour stimulus function)
$\lambda$	wavelength
$\rho(\lambda)$	spectral reflectance
$\tau(\lambda)$	spectral transmittance

## 4 Standard Method

The Standard may be used in conjunction with the CIE 1931 standard colorimetric observer or the CIE 1964 standard colorimetric observer. If the angle subtended at the eye by the colour stimulus (or fields to be matched in colour) is between about  $1^\circ$  and  $4^\circ$  the CIE 1931 standard colorimetric observer shall be used. If this angular subtense is greater than  $4^\circ$  the CIE 1964 standard colorimetric observer shall be used. The same colorimetric observer shall be used for all stimuli to be compared with each other.

### 4.1 Calculation of Tristimulus Values

In the CIE 1931 standard colorimetric system, tristimulus values  $X$ ,  $Y$  and  $Z$  are defined as integrals over the spectral range 360 nm to 830 nm according to the equations:

$$\begin{aligned} X &= k \int_{\lambda} \varphi_{\lambda}(\lambda) \bar{x}(\lambda) d\lambda \\ Y &= k \int_{\lambda} \varphi_{\lambda}(\lambda) \bar{y}(\lambda) d\lambda \\ Z &= k \int_{\lambda} \varphi_{\lambda}(\lambda) \bar{z}(\lambda) d\lambda \end{aligned} \quad (1)$$

where  $\varphi_{\lambda}(\lambda)$  is the colour stimulus function to be evaluated;  $\bar{x}(\lambda)$ ,  $\bar{y}(\lambda)$ ,  $\bar{z}(\lambda)$  are the colour-matching functions of the CIE 1931 standard colorimetric observer; and  $k$  is a normalizing constant defined below. The standard method for evaluating these integrals is numerical summation from 360 nm to 830 nm at wavelength intervals,  $\Delta\lambda$ , equal to 1 nm according to the equations:

$$\begin{aligned} X &= k \sum_{\lambda} \varphi_{\lambda}(\lambda) \bar{x}(\lambda) \Delta\lambda \\ Y &= k \sum_{\lambda} \varphi_{\lambda}(\lambda) \bar{y}(\lambda) \Delta\lambda \\ Z &= k \sum_{\lambda} \varphi_{\lambda}(\lambda) \bar{z}(\lambda) \Delta\lambda \end{aligned} \quad (2)$$

using colour-matching functions  $\bar{x}(\lambda)$ ,  $\bar{y}(\lambda)$ ,  $\bar{z}(\lambda)$  defined with 7 significant figures in ISO 11664-1:2007(E)/CIE S 014-1/E:2006 and a colour stimulus function,  $\varphi_{\lambda}(\lambda)$ , measured using a symmetrical triangular or trapezoidal bandpass with a halfwidth equal to 1 nm.

Tristimulus values are often evaluated on a relative basis. In such cases the relative colour stimulus function,  $\varphi(\lambda)$ , may be used instead of the colour stimulus function,  $\varphi_{\lambda}(\lambda)$ . It is essential that, for stimuli that will be considered together, all the spectral distributions involved be assessed on the same relative basis. The tristimulus values obtained are then relative in the sense that all the values involved may be multiplied by the same single arbitrary constant,  $k$ . In certain cases, however,  $k$  shall be chosen according to agreed conventions; these conventions are explained in 4.2 and 4.3.

**NOTE** The wavelength range of 360 nm to 830 nm is in accordance with established CIE practice (ISO 11664-1:2007(E)/CIE S 014-1/E:2006 and CIE, 2004). Clause 5 of this Standard specifies abridged methods that may be used when data are not available over the full range of 360 nm to 830 nm at 1 nm intervals.

## 4.2 Normalizing Constant for Self-Luminous Light Sources

For self-luminous objects, the normalizing constant,  $k$ , is usually chosen on the grounds of convenience. If, however, in the CIE 1931 standard colorimetric system, the  $Y$  value is required to be numerically equal to the absolute value of a photopic photometric quantity,  $\varphi_\lambda(\lambda)$  shall be the spectral concentration of the radiometric quantity corresponding to the photometric quantity required, and the constant,  $k$ , shall be set equal to  $683 \text{ lm}\cdot\text{W}^{-1}$  which is the numerical value of  $K_m$ , the maximum spectral luminous efficacy in the CIE System of Physical Photometry (ISO 23539:2005(E)/CIE S 010/E:2004).

## 4.3 Normalizing Constant for Reflecting or Transmitting Objects

For reflecting or transmitting object colours, the colour stimulus function,  $\varphi_\lambda(\lambda)$ , shall be replaced by the relative colour stimulus function,  $\varphi(\lambda)$ , evaluated as

$$\varphi(\lambda) = R(\lambda) S(\lambda) \quad (3)$$

$$\text{or} \quad \varphi(\lambda) = \beta(\lambda) S(\lambda) \quad (4)$$

$$\text{or} \quad \varphi(\lambda) = \rho(\lambda) S(\lambda) \quad (5)$$

$$\text{or} \quad \varphi(\lambda) = \tau(\lambda) S(\lambda) \quad (6)$$

where  $R(\lambda)$  is the spectral reflectance factor,  $\beta(\lambda)$  is the spectral radiance factor,  $\rho(\lambda)$  is the spectral reflectance,  $\tau(\lambda)$  is the spectral transmittance and  $S(\lambda)$  is the relative spectral distribution of the illuminant.

In all these cases, the constant,  $k$  shall be chosen so that  $Y = 100$  for objects for which  $R(\lambda)$  (or  $\beta(\lambda)$ ,  $\rho(\lambda)$ ,  $\tau(\lambda)$ ) equals 1 for all wavelengths. Hence

$$k = 100 / \sum_{\lambda} S(\lambda) \bar{y}(\lambda) \quad (7)$$

where the summation range and interval, and the values of  $\bar{y}(\lambda)$ , are the same as in equations (2).

The values of  $Y$  for all objects are then equal to the percentage values of luminous reflectance factor (in the case of  $R(\lambda)$ ), luminance factor (in the case of  $\beta(\lambda)$ ), luminous reflectance (in the case of  $\rho(\lambda)$ ), or luminous transmittance (in the case of  $\tau(\lambda)$ ). This is because the  $\bar{y}(\lambda)$  function is identical to the CIE spectral luminous efficiency function  $V(\lambda)$ .

**NOTE** All four quantities,  $R(\lambda)$ ,  $\beta(\lambda)$ ,  $\rho(\lambda)$  and  $\tau(\lambda)$ , are ratios. If, for convenience, any of these quantities are reported as percentages, the numerical values must be divided by 100 for the above derivation of  $k$  to be correct.

## 4.4 CIE 1964 Standard Colorimetric System

The colour-matching functions  $\bar{x}_{10}(\lambda)$ ,  $\bar{y}_{10}(\lambda)$ ,  $\bar{z}_{10}(\lambda)$  of the CIE 1964 standard colorimetric observer (ISO 11664-1:2007(E)/CIE S 014-1/E:2006) may be used in place of  $\bar{x}(\lambda)$ ,  $\bar{y}(\lambda)$ ,  $\bar{z}(\lambda)$ . In this case, the symbols  $X$ ,  $Y$ ,  $Z$  and  $k$  shall be replaced by  $X_{10}$ ,  $Y_{10}$ ,  $Z_{10}$  and  $k_{10}$  in all equations in this Standard.

NOTE Use of the  $\bar{y}_{10}(\lambda)$  function for photometry (as in 4.2), with the appropriate value of  $K_{m,10}$ , ( $683,6 \text{ lm}\cdot\text{W}^{-1}$ ) has not been standardized or approved by the General Conference on Weights and Measures (Conférence Générale des Poids et Mesures, CGPM). However, CIE Technical Report 165:2005 (CIE, 2005a) has recommended that the function can be so used especially if luminance has to be determined para-foveally.

## 5 Abridged Methods

In some cases, the standard method defined in Clause 4 of this Standard cannot be used because the colour stimulus function or relative colour stimulus function is not available over the full range of 360 nm to 830 nm in 1 nm intervals.

### 5.1 Abridged Method for 5 nm Data

If it is demonstrated that the resulting errors are insignificant for the purpose of the user, tristimulus values  $X$ ,  $Y$ ,  $Z$  shall be calculated by numerical summation from 380 nm to 780 nm at wavelength intervals,  $\Delta\lambda$ , equal to 5 nm according to equations (2) to (7) using colour-matching functions  $\bar{x}(\lambda)$ ,  $\bar{y}(\lambda)$ ,  $\bar{z}(\lambda)$ , defined with 7 significant figures in ISO 11664-1:2007(E)/CIE S 014-1/E:2006. (See 6.3 for bandwidth requirements.)

If the colour stimulus function or relative colour stimulus function data are provided at wavelength intervals of 2 nm, 3 nm or 4 nm, the same method shall be used, subject to the same conditions.

If the wavelength interval is less than 5 nm but not an integer multiple of 1 nm, either the colour-matching functions and the illuminant, or the colour stimulus data must be interpolated so that they match. See 6.2 for guidance on this.

NOTE 1 Some publications give the colour-matching functions from ISO 11664-1:2007(E)/CIE S 014-1/E:2006 with values rounded to 4 significant figures. These rounded values may be used provided that it is demonstrated that the resulting errors are insignificant for the purpose of the user.

NOTE 2 Some CCD array spectrometers record data at unequal wavelength intervals. In this case,  $\Delta\lambda$  in equations (2) and (7) will vary.

### 5.2 Abridged Method for 10 nm or 20 nm Data for Reflecting or Transmitting Objects

This Standard does not cover abridged methods for 10 nm or 20 nm data. It applies only to data at 5 nm intervals or less. A CIE Technical Committee (TC 1-71) is currently working to develop a recommendation in this area.

NOTE 1 A common method for calculating tristimulus values  $X$ ,  $Y$ ,  $Z$  of reflecting or transmitting object colours at wavelength intervals,  $\Delta\lambda$ , equal to 10 nm or 20 nm is to use the following equations:

$$\begin{aligned} X &= \sum_{\lambda} R(\lambda) W_x(\lambda) \\ Y &= \sum_{\lambda} R(\lambda) W_y(\lambda) \\ Z &= \sum_{\lambda} R(\lambda) W_z(\lambda) \end{aligned} \tag{8}$$

where  $R(\lambda)$  is the spectral reflectance factor measured using a symmetrical triangular or trapezoidal bandpass with a halfwidth equal to the wavelength interval (10 nm or 20 nm) and

$W_x(\lambda)$ ,  $W_y(\lambda)$  and  $W_z(\lambda)$  are pre-calculated weighting functions that take into account the colour-matching functions, the relative spectral distribution of the illuminant, the wavelength interval, the bandwidth and the normalizing constant, to give the best fit to the standard method (see Clause 4) based on the assumption that  $R(\lambda)$  varies smoothly between the measured (10 nm or 20 nm interval) values. The spectral reflectance factor  $R(\lambda)$  may be replaced in equations (8) by the spectral radiance factor  $\beta(\lambda)$ , the spectral reflectance  $\rho(\lambda)$ , or the spectral transmittance  $\tau(\lambda)$ .

NOTE 2 Examples of pre-calculated weighting functions prepared for this purpose are given in ASTM E308-08 (ASTM, 2008a).

NOTE 3 Details of the calculation of weighting functions have been published (Fairman, 1985; Venable, 1989; ASTM, 2008b; Li et al., 2004).

### 5.3 Abridged Method for 10 nm or 20 nm Data for Self-Luminous Light Sources

This Standard does not cover abridged methods for 10 nm or 20 nm data. It applies only to data at 5 nm intervals or less. Data intervals of larger than 5 nm should not be used for light sources, except when it is demonstrated that the error for a larger interval is negligible for the particular light source being evaluated. For many self-luminous light sources, particularly those with narrow-band features such as fluorescent lamps, gas discharge lamps and light-emitting diodes, calculation of tristimulus values from stimulus functions measured at wavelength intervals,  $\Delta\lambda$ , greater than 5 nm will not give accurate results.

## 6 Supplementary Treatment of Input Data

This clause outlines supplementary treatment of data necessary to apply the methods of Clauses 4 and 5 or to correct measured data for improved accuracy.

The use of the methods described in Clauses 4 and 5 of this Standard requires that the colour stimulus function,  $\varphi_x(\lambda)$ , or the relative colour stimulus function,  $\varphi(\lambda)$ , be known over a specified wavelength range, at a specified interval, and with a specified bandwidth. It is important to use the same wavelength range, interval and bandwidth throughout for any set of calculations in which data for different colours are to be compared precisely. In practical applications however, all the required data may not be available because the measurement was made at intervals greater than specified, and/or unequal wavelength intervals were used, and/or data at the spectral extremes were omitted, and/or the bandwidth was not equal to the sampling interval, and/or the bandpass shape was not a symmetrical triangle or trapezium. Sometimes it is possible to predict the needed but unmeasured data, although calculation from predicted data may be inexact. Thus, prediction methods should only be used if the user has reviewed the impact on the final results and has demonstrated that the resulting errors are insignificant for the purpose of the user. Some guidance and warnings are given in 6.1, 6.2 and 6.3.

### 6.1 Extrapolation

Extrapolation of measured data if the measurement range is less than 380 nm to 780 nm may cause errors and shall be used only if it can be demonstrated that the resulting errors are insignificant for the purpose of the user.

NOTE 1 When predicting needed values of  $\varphi_x(\lambda)$ ,  $\varphi(\lambda)$ ,  $R(\lambda)$ ,  $\beta(\lambda)$ ,  $\rho(\lambda)$  or  $\tau(\lambda)$  beyond the measured range, unmeasured values may, as a rough approximation, be set equal to the nearest measured value of the appropriate quantity (CIE, 2004) or simple linear extrapolation may be used (CIE, 2005b). Missing values may be given other values, such as zero or 100 % if there is a reason to do so based on the data or other experience (CIE, 2005b).

NOTE 2 The sum of the weights at the unmeasured wavelengths divided by the sum of the weights at all wavelengths is a measure of the maximum error introduced by using predicted values instead of measured ones.

## 6.2 Interpolation

If the wavelengths of the measured data do not match exactly those of the colour matching functions, one or the other need to be interpolated so that they match. For reflectance and transmittance data, and for light sources having smooth spectral distribution curves, either the measured data or the colour matching functions may be interpolated. For interpolation of measured data, an estimate of values between the measured values may be found by a theoretical equation representing the data if such an equation exists, or by mathematical curve fitting. A review and recommendation is given in CIE, 2005b. If the colour matching functions are interpolated, linear interpolation shall be used to interpolate points between the 1 nm intervals (ISO 11664-1:2007(E)/CIE S 014-1/E:2006). For light source data that can include narrowband peaks and emission lines, the measured data shall not be interpolated, and the colour matching functions shall be interpolated to match the wavelengths of the measured data.

NOTE 1 Interpolation of measured data to smaller intervals normally does not improve calculated colour accuracy. The data intervals described in Clauses 4 and 5 apply to the original measured data. If original data interval is much larger than 5 nm, the data do not meet the requirements of this Standard even if they are converted to 5 nm or smaller intervals by interpolation.

NOTE 2 The wavelength interval of instruments is often matched with the instrument bandwidth. In such cases, if the data are interpolated to a different interval, the integrity of the matching will be lost, and the converted data will not be applicable for bandpass corrections such as those referenced in 6.3.

## 6.3 Bandwidth

All spectral data obtained from instruments have finite bandwidth, which will propagate to errors in calculated tristimulus values. Errors arising from bandwidth are generally much larger (by an order of magnitude) than the calculation errors associated with data intervals.

The bandwidth of the instruments used to produce the input data for this Standard shall be 5 nm or less, unless the bandwidth is corrected by one of the available methods such as ASTM, 2009; Fairman, 2010; Gardner, 2006; Kostkowski, 1997; Ohno, 2005; Robertson, 1967; Stearns and Stearns, 1988; Venable, 1989; and Woolliams and Cox, 2005.

For the measurement of light sources having emission lines, such as discharge lamps, the data interval and the bandwidth shall be matched so that any wavelength points will not be under-sampled or over-sampled relative to other wavelength points. For this, the bandpass should be a symmetrical triangular shape, and its half-bandwidth should be matched with the wavelength interval or an integer multiple thereof. This matching condition is not critical if the interval is much smaller than the bandwidth. This matching condition is not required for reflectance/transmittance measurements of object colours, because these spectra are relatively smooth. An exception is, however, when a bandpass correction is to be applied, in which case the matching condition is required.

## 7 Chromaticity Coordinates

If required, chromaticity coordinates  $x$ ,  $y$  and  $z$  shall be calculated from the tristimulus values  $X$ ,  $Y$  and  $Z$  as follows:

$$\begin{aligned}x &= \frac{X}{X + Y + Z} \\y &= \frac{Y}{X + Y + Z} \\z &= \frac{Z}{X + Y + Z}\end{aligned}\tag{9}$$

Because of the relation  $x + y + z = 1$ , it suffices to quote  $x$  and  $y$  only.

The diagram produced by plotting  $x$  as abscissa and  $y$  as ordinate shall be referred to as the CIE 1931 chromaticity diagram or the CIE  $(x, y)$  diagram.

The chromaticity coordinates  $x_{10}$ ,  $y_{10}$  and  $z_{10}$  shall be computed similarly from  $X_{10}$ ,  $Y_{10}$  and  $Z_{10}$ , and the diagram produced by plotting  $x_{10}$  as abscissa and  $y_{10}$  as ordinate shall be referred to as the CIE 1964 chromaticity diagram or the CIE  $(x_{10}, y_{10})$  diagram.

## 8 Numerical Procedures

All numerical calculations shall be carried out using the full number of significant digits provided by the input data. Final results shall be rounded to the number of significant digits indicated by the uncertainty of the measurements.

## 9 Presentation of Results

Any reporting of tristimulus values and derived parameters shall be accompanied by a statement concerning the measurement geometry, observer, illuminant (for object colours), and sample backing (for reflecting object colours), used to generate them. The report shall also specify the wavelength range and the interval of the summation.