AN AMERICAN NATIONAL STANDARD

ENGINEERING DRAWING AND RELATED DOCUMENTATION PRACTICES

Mathematical Definition of Dimensioning and Tolerancing Principles

ASME Y14.5.1M-1994

The American Society of Mechanical Engineers

345 East 47th Street, New York, N.Y. 10017

Date of Issuance: January 31, 1995

This Standard will be revised when the Society approves the issuance of a new edition. There will be no addenda or written interpretations of the requirements of this Standard issued to this Edition.

FOREWORD

(This Foreword is not a part of ASME Y14.5.1M-1994.)

The Y14 Committee created the Y14.5.1 Subcommittee in response to a need identified during a National Science Foundation (NSF) workshop. The International Workshop on Mechanical Tolerancing was held in Orlando, Florida, in late 1988. The workshop report strongly identified a need for a mathematical definition for the current tolerancing standards. Tom Charlton coined the phrase "mathematization of tolerances." The phrase meant to add mathematical rigor to the Y14.5M standard. The response is the present standard, ASME Y14.5.1M-1994. This new standard creates explicit definitions for use in such areas as Computer Aided Design (CAD) and Computer Aided Manufacturing (CAM).

The Committee has met three times a year since their first meeting of January of 1989 in Long Boat Key, Florida. Initial discussions covered scope of the document, boundary definitions, size, and datums.

The Committee identified four major divisions of a tolerance: 1) the mathematical definition of the tolerance zone; 2) the mathematical definition of conformance to the tolerance; 3) the mathematical definition of the actual value; 4) the mathematical definition of the measured value. The Subcommittee later decided that the measured value was beyond the scope of this Standard. When this Standard defines part conformance, it consists of the infinite set of points that make up all the surfaces of the part, and it is addressing imperfect form semantics.

This Standard does not fully address the issue of boundary, that is where one surface stops and the other surface starts. The Subcommittee hopes to define this in the next edition of this Standard.

The definition of size took up many days of discussions and interaction with the Y14.5 Subcommittee. It always came back to the statement of a micrometer-type two point cross-sectional measurement. The difficulty comes from the method of defining the cross-section. Consider a figure such as an imperfectly formed cylinder. When considering the infinite set of points that make up the surface, what is the intent behind a two point measurement? Most of the reasons appear to be for strength. Yet, a two point cross-sectional definition doesn't define strength on, for instance, a three-lobed part. These and other considerations led to the existing definition. The pictorial definition, presented in Section 2, is the smallest of the largest elastic perfect spheres that can be passed through the part without breaking the surface. This Standard does not address measurement, yet often a two point cross-sectional measurement is adequate for form, fit, and function.

The subject of datums also led to many hours of work by the Subcommittee. The current definitions, presented in Section 4, were the result of evaluating a number of approaches against four criteria: 1) conformance to Y14.5M; 2) whether a unique datum is defined; 3) whether the definition is mathematically unambiguous; and 4) whether the definition conveys design intent. A fifth criterion, whether the definition was measurable, was not used for reasons discussed above. The end result of this work was based on feedback from the Y14.5M Subcommittee when Y14.5.1 presented its analysis, and involved a change in its thinking about datums. The initial view of a datum was as something established before a part feature is evaluated. The current definitions involve a different view that a datum exists *for the sake of the features* related to it. The result was a consolidation of the issues involved with "wobbling" datums and the issues involved with datum features of size at MMC or LMC. These apparently dissimilar issues are unified mathematically in the concepts of "candidate datum" and "candidate datum reference frame."

A special thanks to the Y14 Main Committee and Y14.5 Subcommittee members in their support and encouragement in the development of this Standard.

Also of note are the participation and contributions of Professor Ari Requicha of University of Southern California, Professor Josh Turner of Rensselaer Polytechnic Institute, and Professor Herb Voelcker of Cornell University.

Suggestions for improvement of this Standard are welcome. They should be sent to the American Society of Mechanical Engineers, Att: Secretary, Y14 Main Committee, 345 East 47th Street, New York, NY 10017.

This Standard was approved as an American National Standard on November 14, 1994.

ASME STANDARDS COMMITTEE Y14
Engineering Drawing and Related Documentation Practices

(The following is the roster of the Committee at the time of approval of this Standard.)

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ENGINEERING DRAWING AND RELATED DOCUMENTATION PRACTICES

MATHEMATICAL DEFINITION OF DIMENSIONING AND TOLERANCING PRINCIPLES

1 SCOPE AND DEFINITIONS

1.1 General

This Standard presents a mathematical definition of geometrical dimensioning and tolerancing consistent with the principles and practices of ASME Y14.5M-1994, enabling determination of actual values. While the general format of this Standard parallels that of ASME Y14.5M-1994, the latter document should be consulted for practices relating to dimensioning and tolerancing for use on engineering drawings and in related documentation.

Textual references are included throughout this Standard which are direct quotations from ASME Y14.5M-1994. All such quotations are identified by italicized type. Any direct references to other documents are identified by an immediate citation.

The definitions established in this Standard apply to product specifications in any representation, including drawings, electronic exchange formats, or data bases. When reference is made in this Standard to a part drawing, it applies to any form of product specification.

1.1.1 Units. The International System of Units (SI) is featured in the Standard because SI units are expected to supersede United States (U.S.) customary units specified on engineering drawings.

1.1.2 Figures. The figures in this Standard are intended only as illustrations to aid the user in understanding the principles and methods described in the text. In some instances figures show added detail for emphasis; in other instances figures are incomplete by intent. Any numerical values of dimensions and tolerances are illustrative only.

1.1.3 Notes. Notes shown in capital letters are intended to appear on finished drawings. Notes in lower case letters are explanatory only and are not intended to appear on drawings.

1.1.4 Reference to Gaging. This Standard is not intended as a gaging standard. Any reference to gaging is included for explanatory purposes only.

1.2 References

When the following American National Standards referred to in this Standard are superseded by a revision approved by the American National Standards Institute, the revision shall apply.

ANSI B46.1-1985, Surface Texture

ASME Y14.5M-1994, Dimensioning and Tolerancing

1.3 Mathematical Notation

This Subsection describes the mathematical notation used throughout this Standard, including symbology (typographic conventions) and algebraic notation.

1.3.1 Symbology. All mathematical equations in this Standard are relationships between real numbers, three-dimensional vectors, coordinate systems associated with datum reference frames, and sets of these quantities. The symbol conventions shown in Table 1.3 are used for these quantities.

These symbols may be subscripted to distinguish between distinct quantities. Such subscripts do not change the nature of the designated quantity.

Technically, there is a difference between a vector and a vector with position. Generally in this Standard, vectors do not have location. In particular, direction vectors, which are often defined for specific points on curves or surfaces, are functions of position on the geometry, but are not located at those points. (Another conventional view is that all vectors are located at the origin.) Throughout this Standard, position vectors are used to denote points in space. While there is a technical difference between a vector

TABLE 1.3.1 SYMBOLOGY

Quantity	Symbol
Real Numbers	Plain-face, italic, lower-case English or lower-case Greek letters (t, r, θ, etc.)
Vectors	Bold-face, italic English letters with an arrow diacritical mark ($\vec{T}$, etc.)
Unit Vectors	Bold-face, italic English letters with a carat diacritical mark ($\hat{N}$, etc.)
Functions (real or vector-valued)	A real number or vector symbol (depending on the kind of value of the function) followed by the parameters of the function in parentheses [$r(\vec{P})$, etc.]
Datum Reference Frames (coordinate systems)	Plain-face, upper case Greek letter (Γ, etc.)
Sets	Plain-face, italic, upper-case English letters (S, F, etc.)

and a point in space, the equivalence used in this Standard should not cause confusion.

1.3.2 Algebraic Notation. A vector can be expanded into scalar components (with the components distinguished by subscripts, if necessary). Let $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ be the unit vectors along the x, y, and z axes, respectively, of a coordinate system. Then a vector $\vec{V}$ can be uniquely expanded as:

$$\vec{V} = a\,\hat{\imath} + b\,\hat{\jmath} + c\,\hat{k}$$

The vector can be written $\vec{V} = (a,b,c)$. The magnitude (length) of vector $\vec{V}$ is denoted by $|\vec{V}|$ and can be evaluated by:

$$|\vec{V}| = \sqrt{a^2 + b^2 + c^2}$$

A unit vector $\vec{V}$ is any vector with magnitude equal to one. The scalar product (dot product; inner product) of two vectors $\vec{V}_1 = (a_1, b_1, c_1)$ and $\vec{V}_2 = (a_2, b_2, c_2)$ is denoted by $\vec{V}_1 \cdot \vec{V}_2$. The scalar product is a real number given by:

$$\vec{V}_1 \cdot \vec{V}_2 = a_1a_2 + b_1b_2 + c_1c_2$$

and is equal in value to the product of the lengths of the two vectors times the cosine of the angle between them. The vector product (cross product; outer product) of two vectors $\vec{V}_1$ and $\vec{V}_2$ is denoted by $\vec{V}_1 \times \vec{V}_2$. The cross product is a vector $\vec{V}_3 = (a_3, b_3, c_3)$ with components given by:

$$a_3 = b_1c_2 - b_2c_1$$
$$b_3 = a_2c_1 - a_1c_2$$
$$c_3 = a_1b_2 - a_2b_1$$

The magnitude of the cross product is equal in value to the product of the lengths of the two vectors times the sine of the angle between them.

For a given feature, the notation $r(\vec{P}, \Gamma)$ will denote the distance from a point $\vec{P}$ to true position (see Subsection 1.4) in datum reference frame Γ. When the datum reference frame is understood from the context, the notation $r(\vec{P})$ will be used. Figure 1-1 shows a case of a true position axis. If the axis is represented by a point $\vec{P}_0$ on the axis and a direction $\hat{N}$ (a unit vector), then $r(\vec{P})$ can be evaluated by either of the following formulas:

$$r(\vec{P}) = \sqrt{|\vec{P} - \vec{P}_0|^2 - [(\vec{P} - \vec{P}_0) \cdot \hat{N}]^2}$$

or

$$r(\vec{P}) = |(\vec{P} - \vec{P}_0) \times \hat{N}|$$

The first equation is a version of the Pythagorean Theorem. The second equation is based on the properties of the cross product.

1.4 DEFINITIONS

The following terms are defined as their use applies to this Standard. ASME Y14.5M-1994 should be consulted for definitions applying to dimensioning and tolerancing.

1.4.1 Actual mating surface. A surface of perfect form which corresponds to an actual part feature. For a cylindrical or spherical feature, the actual mating surface is the actual mating envelope. For a planar feature, it is defined by the procedures defining a primary datum plane.

1.4.2 Actual value. A unique numerical value representing a geometric characteristic associated

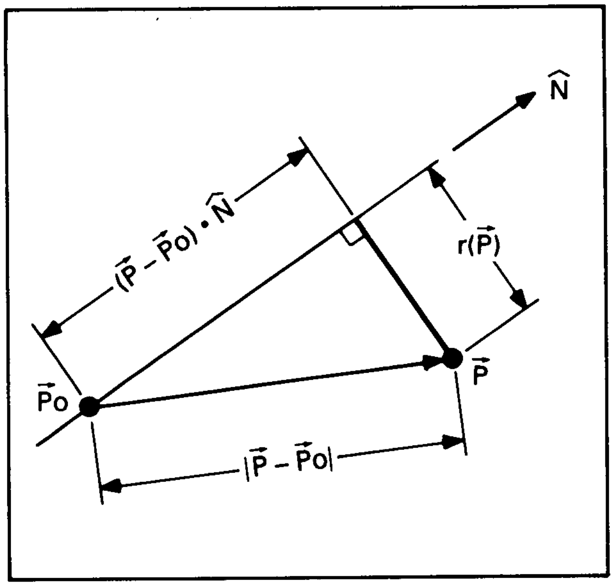


FIG. 1-1 THE DISTANCE FROM A POINT TO A LINE

with a workpiece. Example characteristics are flatness, perpendicularity, position, size, etc. Later sections of this Standard give rules for the determination of actual values for specific characteristics.

1.4.3 Candidate datum. A datum established from a datum feature.

1.4.4 Candidate datum reference frame. A datum reference frame established from candidate datums.

1.4.5 Candidate datum reference frame set. The set of all candidate datum reference frames established from a set of referenced datums.

1.4.6 Candidate datum set. The set of all candidate datums that can be established from a datum feature.

1.4.7 Conformance. Applied to a part feature, that condition in which the feature does not violate the constraints defined by the tolerance. For tolerances that reference datums, if the feature does not violate the constraints defined by the tolerance for at least one datum reference frame in the candidate datum reference frame set, then the part feature is in conformance to the tolerance.

1.4.8 Cutting plane. A plane used to establish a planar curve in a feature. The curve is the intersection of the cutting plane with the feature.

1.4.9 Cutting vector. A unit vector on the actual mating surface which, together with the normal to the actual mating surface, defines the direction of the cutting plane. Cutting vectors will be designated $\hat{C}_V$.

1.4.10 Direction vector. A unit vector. Conventionally, directions are associated with various geometries as follows. The direction vector of a straight line (or pair of parallel lines) is parallel to the line(s). The direction vector of a plane (or a pair of parallel planes) is normal to the plane. The direction vector of a cylinder is the direction vector of the cylinder axis.

1.4.11 Element, circular surface. Given an axis and a surface of revolution about that axis, a circular surface element is a set of points determined by the intersection of the given surface with one of the following, as applicable:

(a) a plane perpendicular to the given axis
(b) a cone with axis coincident with the given axis
(c) a cylinder with axis coincident with the given axis

1.4.12 Element, surface line. The intersection between an actual surface and a cutting plane.

1.4.13 Envelope, actual mating. A surface, or pair of parallel surfaces, of perfect form, which correspond to an actual part feature of size, as follows:

(a) For an External Feature. A similar perfect feature counterpart of smallest size which can be circumscribed about the feature so that it just contacts the surface.

(b) For an Internal Feature. A similar perfect feature counterpart of largest size which can be inscribed within the feature so that it just contacts the surface.

Figure 1-2 illustrates the definition of both actual mating envelope and actual minimum material envelope (see definition below) for both internal and external features.

In certain cases (e.g., a secondary datum cylinder) the orientation or position of an actual mating envelope may be restricted in one or more directions. (See Fig. 1-3.)

1.4.14 Envelope, actual minimum material. A surface, or pair of parallel surfaces, of perfect form which correspond to an actual part feature of size, as follows:

(a) For an External Feature. A similar perfect feature counterpart of largest size which can be inscribed within the feature so that it just contacts the surface.

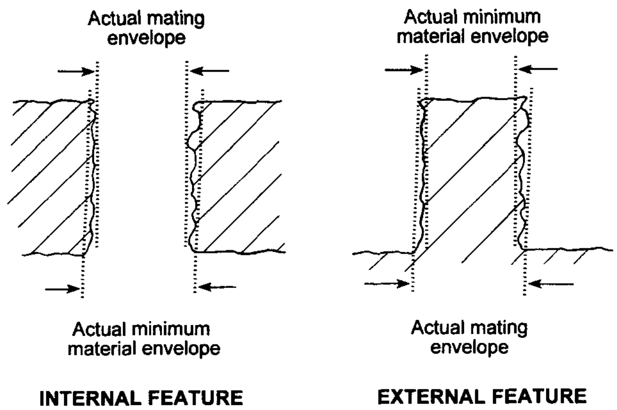


FIG. 1-2 ILLUSTRATION OF ACTUAL MATING ENVELOPE AND ACTUAL MINIMUM MATERIAL ENVELOPE

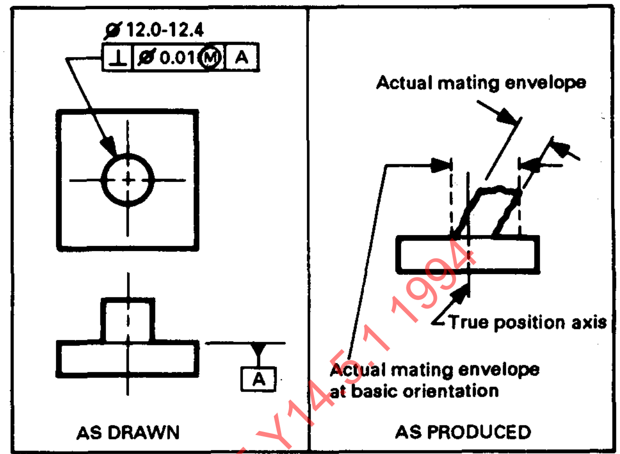


FIG. 1-3 ACTUAL MATING ENVELOPE AND ACTUAL MATING ENVELOPE AT BASIC ORIENTATION

(b) *For an Internal Feature.* A similar perfect feature counterpart of smallest size which can be circumscribed about the feature so that it just contacts the surface.

Figure 1-2 illustrates the definition of both actual mating envelope (see definition below) and actual minimum material envelope for both internal and external features.

In certain cases the orientation or position of the actual minimum material envelope may be restricted in one or more directions.

1.4.15 Mating surface normal. For a given point on a part feature, the direction vector of a line passing through the point and normal to the actual mating surface at the point of intersection of the line with the actual mating surface. (See Fig. 1-4.)

1.4.16 Perfect form. A geometric form that corresponds to the nominal (design) geometry except for possible variations in size, position, or orientation.

1.4.17 Resolved geometry. A collective term for the center point of a sphere, the axis of a cylinder, or the center plane of a width. Conceptually, the resolved geometry of a feature of size is a corresponding feature of size having perfect form and zero size.

1.4.18 Set of support. For a planar feature, a plane that contacts the feature at one or more points, such that the feature is not on both sides of the plane. The concept of set of support is illustrated in Fig. 1-5.

1.4.19 Size, actual mating. The dimension of the actual mating envelope. (The dimension may be a radius or diameter for a spherical or cylindrical

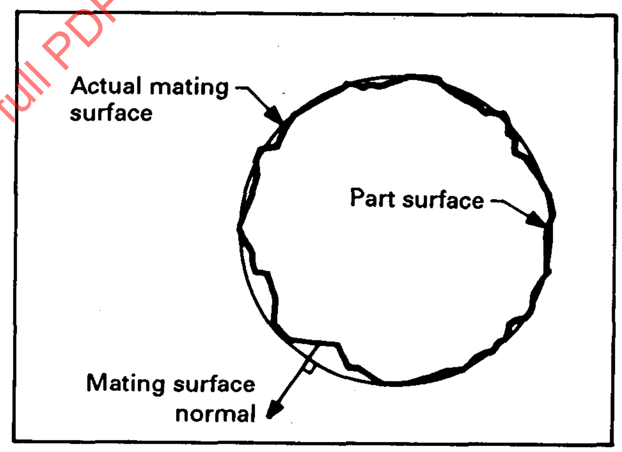


FIG. 1-4 ILLUSTRATION OF MATING SURFACE NORMAL

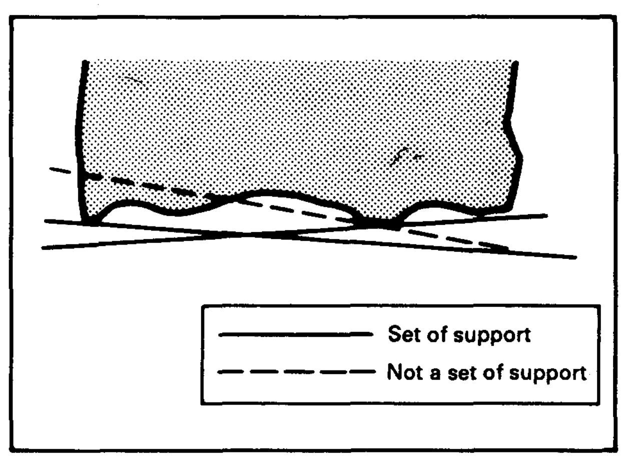


FIG. 1-5 EXAMPLES OF SET OF SUPPORT

envelope, or width or half-width for a parallel-plane envelope, depending on the context. The radius of a cylindrical or spherical actual mating envelope will be designated r_{AM}.)

1.4.20 Size, actual minimum material. The dimension of the actual minimum material envelope. (The dimension may be a radius or diameter for a spherical or cylindrical envelope, or width or half-width for a parallel-plane envelope, depending on the context. The radius of a cylindrical or spherical actual minimum material envelope will be designated r_{AMM}.)

1.4.21 Size, true position mating. The size, optimized over the candidate datum reference frame set, *R*, of the actual mating envelope constrained to be located and oriented at true position. The true position mating size will be designated r_{TP}. If $r(\vec{P}, \Gamma)$ is the distance from a point $\vec{P}$ to the true position in datum reference frame Γ (an element of the candidate datum reference frame set), and if *F* is the feature, then the true position mating size is given by:

$$r_{TP} = \begin{cases} \max\limits_{\Gamma \epsilon R} \min\limits_{\vec{P} \epsilon F} r(\vec{P},\Gamma) & \text{internal features} \\ \min\limits_{\Gamma \epsilon R} \max\limits_{\vec{P} \epsilon F} r(\vec{P},\Gamma) & \text{external features} \end{cases}$$

For more information, see Subsection 4.2.

1.4.22 Size, true position minimum material. The size, optimized over the candidate datum reference frame set, *R*, of the actual minimum material envelope constrained to be located and oriented at true position. The true position minimum material size will be designated r_{TPMM}. If $r(\vec{P},\Gamma)$ is the distance from a point $\vec{P}$ to the true position in datum reference frame Γ (an element of the candidate datum reference frame set), and if *F* is the feature, then the true position minimum material size is given by:

$$r_{TPMM} = \begin{cases} \min\limits_{\Gamma \epsilon R} \max\limits_{\vec{P} \epsilon F} r(\vec{P},\Gamma) & \text{internal features} \\ \max\limits_{\Gamma \epsilon R} \min\limits_{\vec{P} \epsilon F} r(\vec{P},\Gamma) & \text{external features} \end{cases}$$

For more information, see Subsection 4.2.

1.4.23 Spine. A point, simple (non self-intersecting) curve, or simple surface. Spines are used in the definitions of size and circularity.

1.4.24 True position. The theoretically exact position of a feature in a particular datum reference frame. In some contexts, the term "true position" refers to the resolved geometry by which the feature is located.

1.5 SUMMARY OF CONVENTIONAL DESIGNATIONS

Throughout this Standard, conventional designations are used for various quantities. This Subsection summarizes these conventions.

$\hat{C}_P$ = direction vector of a cutting plane
$\hat{C}_V$ = cutting vector
$\hat{D}_1$ = direction vector of the primary datum plane
$\hat{D}_2$ = direction vector of the secondary datum plane
$\hat{D}_3$ = direction vector of the tertiary datum plane
$\hat{N}$ = direction vector of the surface normal
$\vec{P}$ = position vector
$r(\vec{P},\Gamma)$ = the distance of a point $\vec{P}$ to true position in datum reference frame Γ
$r(\vec{P})$ = the distance of a point $\vec{P}$ to true position, in the case that the datum reference frame is understood from context
r_{AM} = actual mating size (radius)
r_{AMM} = actual minimum material size (radius)
r_{TP} = true position mating size (radius)
r_{TPMM} = true position minimum material size (radius)
$\hat{T}$ = direction vector of a tolerance zone
t = an unspecified tolerance value
t_0 = a specific tolerance given on a drawing or part specification
Γ = a candidate datum reference frame

1.6 FORMAT

The format used in this Standard for explanation of geometric characteristics is as follows:

Definition — *narrative and mathematical description of the tolerance zone*

Conformance — *mathematical definition of the conformance*

Actual value — *mathematical definition of actual value*

2 GENERAL TOLERANCING AND RELATED PRINCIPLES

2.1 FEATURE BOUNDARY

Tolerances are applied to features of a part. Generally, features are well-defined only in drawings and computer models. This Section establishes the rules for identifying features on actual parts. Two steps are involved: establishing the surface points of the part and establishing the feature boundary.

2.1.1 Establishing the Surface Points. A certain amount of smoothing of the part surface is implied in this Standard. This smoothing is necessary to distinguish dimensional from roughness, surface texture, material microstructure, and even smaller-scale variations of the part. This Standard incorporates by reference the smoothing functions defined in ANSI B46.1-1985, Surface Texture. These functions define how the variations of physical part surfaces must be smoothed to distinguish dimension and form variation from smaller-scale variations. When reference is made in this Standard to points on a feature, it refers to points on the surface that result after smoothing.

2.1.2 Establishing Feature Boundaries. It is possible to subdivide the boundary of a part in many ways. No rules for establishing a unique subdivision are provided. Any subdivision of an actual part surface into features (subject to some mild limitations such as having one-to-one correspondence with the nominal part features, and preserving adjacency relationships) which favors conformance to all applicable tolerances is acceptable. While tolerance requirements may be simultaneous or independent, the subdivision of a part surface into features cannot vary from one tolerance to another.

2.2 DIMENSION ORIGIN

When a dimension origin is specified for the distance between two features, the feature from which the dimension originates defines an origin plane for defining the tolerance zone. In such cases, the origin plane shall be established using the same rules as are provided for primary datum features. See Section 4, Datum Referencing.

2.3 LIMITS OF SIZE

A feature of size is one cylindrical or spherical surface, or a set of two opposed elements or opposed parallel surfaces, associated with a size dimension. A feature of size may be either an internal feature or an external feature. This Section establishes definitions for the size limits, conformance, and actual value of size for such features. Subject to Rule #1 of ASME Y14.5M-1994, size limits also control form variation. The method by which Rule #1 is applied is discussed below in para. 2.3.2, Envelope Principle. For the definition of form controls, refer to Sections 5 and 6.

2.3.1 Variation of Size

(a) Definition. A size tolerance zone is the volume between two half-space boundaries, to be described below. The tolerance zone does not have a unique form. Each half-space boundary is formed by sweeping a ball of appropriate radius along an acceptable spine, as discussed below. The radii of the balls are determined by the size limits: one ball radius is the least-material condition limit (r_{LMC}) and one is the maximum-material condition limit (r_{MMC}).

A 0-dimensional spine is a point, and applies to spherical features. A 1-dimensional spine is a simple (non self-intersecting) curve in space, and applies to cylindrical features. A 2-dimensional spine is a simple (non self-intersecting) surface, and applies to parallel-plane features. These three types of spines can be more rigorously defined, respectively, as connected regular (in the relative topology) subsets of d-manifolds, for d = 0, 1, and 2. A d-dimensional spine will be denoted as S^d. Also, a (solid) ball of radius r will be denoted as B_r.

A solid $G(S^d,B_r)$ is obtained by sweeping the ball B_r so that its center lies in S^d. $G(S^0,B_r)$ is a single ball bounded by a sphere. If S^1 is a line segment, $G(S^1,B_r)$ is a solid bounded by a cylindrical surface and two spherical end caps. If S^2 is a planar patch,

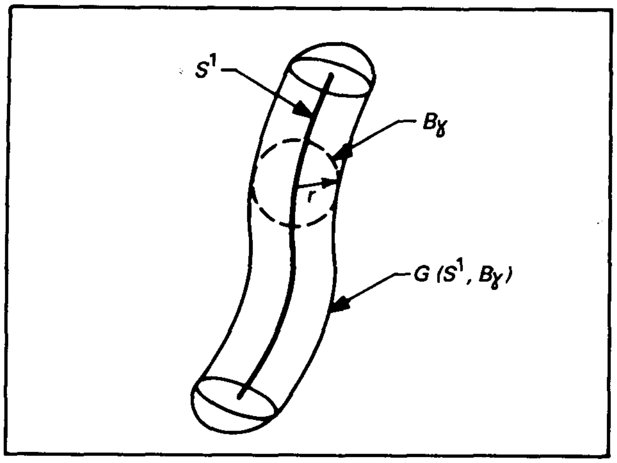
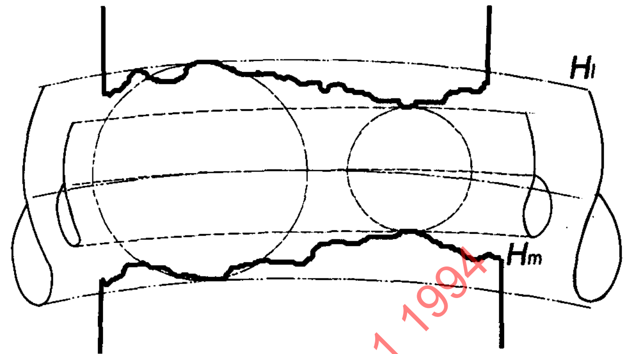


FIG. 2-1 SYMBOLS USED IN THE DEFINITION OF SIZE



When perfect form at MMC not required

FIG. 2-2 CONFORMANCE TO LIMITS OF SIZE

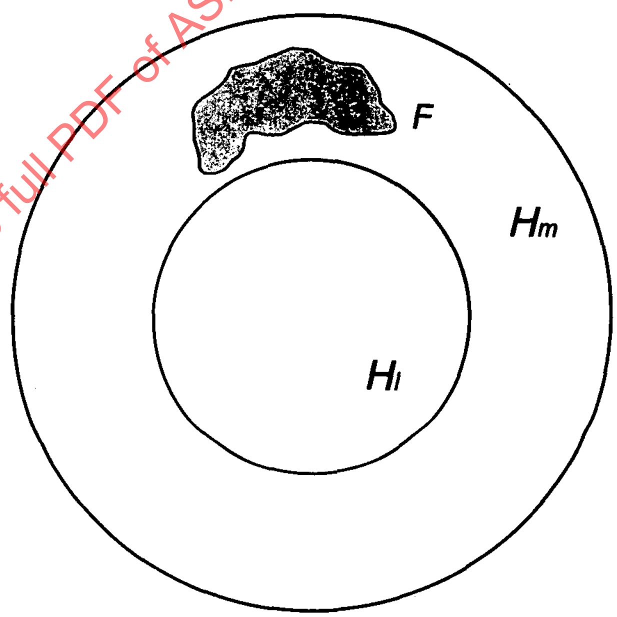


FIG. 2-3 VIOLATION OF THE SURROUND CONDITION FOR AN EXTERNAL FEATURE

$G(S^2, B_r)$ is a solid bounded by two planar patches and some *canal surfaces*. (Canal surfaces are obtained by sweeping spheres, or balls, so that their centers lie on a curve in space.) Figure 2-1 shows a one-dimensional spine and its associated solid for a ball of radius r.

S^1 and S^2 need not be portions of lines or planes, respectively. If necessary, S^1 or S^2 can be extended to infinity, or closed upon itself, so that the resulting solid G is a half-space. The spine, along with the balls, also defines the symmetric axis transformation of such solids.

(b) Conformance. A feature of size, F, conforms to the limits of size r_{LMC} and r_{MMC} if there exist two spines, S_l corresponding to r_{LMC} and S_m corresponding to r_{MMC}, and two associated solids, $G_l = G(S_l, Br_{LMC})$ and $G_m = G(S_m, Br_{MMC})$, that satisfy three conditions described below.

If F is an external feature, then let $H_l = G_l$ and $H_m = G_m$. If F is an internal feature, then let H_l be the complement of G_l and H_m be the complement of G_m. (See Fig. 2-2.) F conforms to its limits of size if:

- $H_l \subset H_m$
- $F \subset H_m - H_l$
- F surrounds (encloses) the boundary of H_l (if F is an external feature) or the boundary of H_m (if F is an internal feature).

The purpose of the third condition is illustrated by Fig. 2-3. The figure shows a configuration where an external feature F satisfies the first two conditions of conformance but does not surround the boundary of H_l.

(c) Actual value. Two actual values are defined. The actual external (to the material) size of an external (respectively, internal) feature is the smallest (respectively, largest) size of the ball to which the feature conforms. The actual internal size is the largest (respectively, smallest) size of the ball to which the feature conforms. The size may be expressed as a radius or diameter, as appropriate to the application.

2.3.2 Variation of Size Under the Envelope Principle

(a) Definition. If Rule #1 applies, the MMC limit of size establishes a boundary of perfect form (envelope) at MMC. The tolerance zone for a feature of size under the envelope principle is as described in the previous section, with the further constraint that

S_m is of perfect form (a straight line for cylindrical features or a plane for parallel plane features).

(b) Conformance. A feature conforms to the envelope principle if it conforms to a size tolerance zone with S_m a perfect-form spine.

(c) Actual value. Two actual values are defined. The actual external (to the material) size of an external (respectively, internal) feature is the smallest (respectively, largest) diameter of the ball to which the feature conforms. The actual internal size is the largest (respectively, smallest) diameter of the ball to which the feature conforms. The boundary associated with S_m for the actual external size is the actual mating envelope. Note that under the envelope principle, the actual external size is identical to the actual mating size; however, the actual internal size is not the actual minimum material size because the boundary associated with S_l is not of perfect form.

3 SYMBOLOGY

There are no concepts in Section 3 of ASME Y14.5M-1994 that require mathematical definition.

4 DATUM REFERENCING

4.1 GENERAL

This Section contains mathematical methods for establishing datums from datum features and for establishing datum reference frames. Datum reference frames are coordinate systems used to locate and orient part features. Constructing a datum reference frame is a two step process. The first step is to develop datum(s) from one or more datum features on the part. The second step is to determine the position and orientation of the datum reference frame from the datums. These steps are described in Subsections 4.3 and 4.4, respectively. Subsection 4.2 discusses basic concepts of datum referencing.

4.2 CONCEPTS

The concept of datums established within ASME Y14.5M-1994 refers to "processing equipment" such as "machine tables and surface plates" that are used to simulate datums for applications such as measurement. This present Standard establishes mathematical concepts for datums and datum reference frames. It considers all points on each feature, and deals with imperfect-form features. The attempt here is to refine the concepts of ASME Y14.5M-1994 by establishing a mathematical model of perfect planes, cylinders, axes, etc. that interact with the infinite point set of imperfectly-formed features. This Standard defines the datums that are simulated by processing equipment.

In this Standard, the part is assumed to be fixed in space and the datums and datum reference frames are established in relation to the part. This approach can be contrasted to that of ASME Y14.5M-1994, where the datums and datum reference frames are assumed to be fixed in space and the part is moved into the datum reference frame. The two approaches are different, but the end results are identical.

Multiple valid datums may be established from a datum feature. This may happen, for example, if the feature "rocks" or if a datum feature of size is referenced at MMC and manufactured away from MMC size. The set of datums that can be established from a datum feature is called a *candidate datum set*. Since a datum feature may generate more than one datum, multiple datum reference frames may exist for a single feature control frame. (Or, from the viewpoint of ASME Y14.5M-1994, the part may move in the datum reference frame.) The set of datum reference frames established from one or more datum features is called a *candidate datum reference frame set*.

For tolerances that reference datums, if the feature does not violate the constraints defined by the tolerance for at least one datum reference frame in the candidate datum reference frame set, then the part feature is in conformance to the tolerance. There is a candidate actual value associated with each candidate datum reference frame in the candidate datum reference frame set. The actual value associated with the tolerance is the minimum candidate actual value.

4.3 ESTABLISHING DATUMS

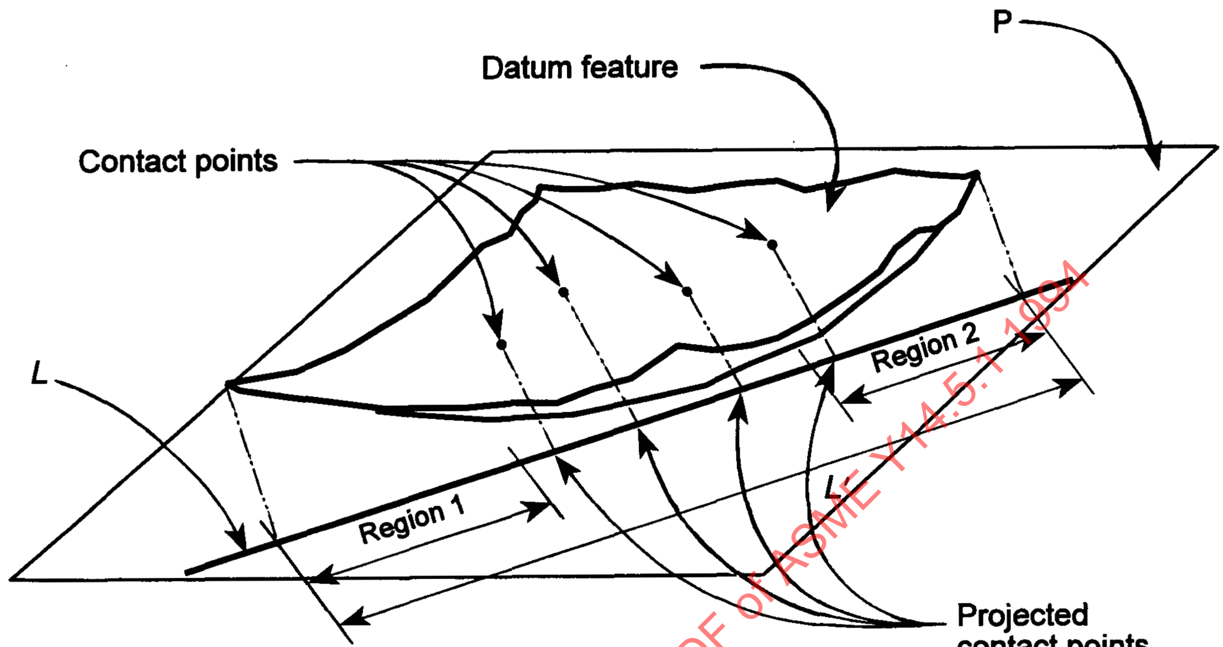
The method of establishing datums depends on the type of datum feature (flat surface, cylinder, width, or sphere), the datum precedence (primary, secondary, or tertiary) in the feature control frame, and (for datum features of size) the material condition of the datum reference.

4.3.1 List of Datum Feature Types. The following classification of datum features is used in this Section:

- Datum features not subject to size variation:
  - Planes
- Datum features subject to size variation (Cylinders, Widths, Spheres):
  - Features referenced at RFS
  - Features referenced at MMC
  - Features referenced at LMC

This Standard does not specify how to establish datums for screw threads, gears, splines, or mathematically defined surfaces (such as sculptured surfaces).

4.3.2 Planar Datum Features. The candidate datum set for a nominally flat datum feature is defined in a procedural manner. This empirical defini-



P — candidate datum plane
Contact points — points where the datum feature contacts *P*
L — any line on plane *P*
L′ — line segment corresponding to the projection of the datum feature on *L*
Regions 1, 2 — regions at the end of *L′*, each with length *xL′* (default *x*=⅓)

FIG. 4-1 CONSTRUCTION FOR TESTING WHETHER A PLANE IS A VALID DATUM PLANE

tion specifies a set of datums which are reasonable from a functional standpoint. If the datum feature is perfectly flat, the candidate datum set consists of only one datum; otherwise it may consist of more than one datum. This is equivalent to "rocking" the datum feature on a perfect surface plate. The definition below limits the amount that the datum feature can "rock" in a manner that is roughly proportional to the form variation of the datum feature.

(a) Primary Planar Datum Features. The candidate datum set for a nominally flat primary datum feature is defined by the following procedure:

(1) Consider a plane *P* which is an external set of support for the datum feature. Let *C* be the set of contact points of the datum feature and *P*.

(2) Consider an arbitrary line *L* in *P*. Orthogonally project each point on the boundary of the datum feature onto *L*, giving the line segment *L′*. Consider regions of *L′* that are within some fraction *x* of the endpoints of *L′*. That is, if the length of *L′* is *n*, consider regions of *L′* within a distance *xn* of the endpoints of *L′*. Unless otherwise specified on the drawing, the value of *x* shall be 1/3. If all of the orthogonal projections of the points in *C* are within either single region, then plane *P* is rejected as a valid datum plane.

(3) Do this for all lines in *P*. (Note that parallel lines will yield identical results.) If no line rejects *P*, then *P* is a candidate datum for the datum feature.

The procedure is illustrated in Fig. 4-1, which shows one line direction *L*. The line segment *L′* is bounded by the projection of the datum feature onto *L*. The particular line direction illustrated in the figure does not reject *P* as a valid datum plane since the projections of the contact points are not all in region 1 or all in region 2 of *L′*. Note that only the direction of *L* in the plane *P* is important. *P* is a candidate datum for the datum feature if it is not rejected by any line direction in *P*.

(b) Secondary Planar Datum Features. The candidate datum set for a secondary planar datum feature is determined by one of the following:

(1) If the primary datum is a point, use the procedure for a primary planar datum feature to establish the secondary datum.

(2) If the primary datum is an axis nominally normal to the secondary datum, then, for each candidate datum in the candidate datum set for the primary datum feature, the candidate datum set for the secondary datum feature includes the unique plane which is basically oriented relative to that primary datum and which forms a set of support for the secondary datum feature.

(3) If neither (1) nor (2) apply, use the procedure for a primary planar datum feature modified in the following ways: Given a primary datum from the primary candidate datum set, each plane *P* being considered as a secondary datum must be basically oriented (not located) to the primary datum. Also, each line *L* in *P* being considered must be perpendicular to the direction vector of the primary datum. (Only one line in *P* must be considered.)

(c) Tertiary Planar Datum Features. If the first two datums leave a rotational degree of freedom (see Subsection 4.4), then the candidate datum set is formed by the procedure for a primary planar datum feature modified such that each plane *P* being considered must be basically oriented (but not necessarily basically located) relative to the datums of higher precedence, and one line *L* is to be considered, which must be perpendicular to the axis established by the higher precedence datums. If the first two datums do not leave a rotational degree of freedom, the candidate datum set consists of the plane which is basically oriented (but not necessarily basically located) relative to the datums of higher precedence and which forms a set of support for the datum feature.

4.3.3 Datum Features Subject to Size Variation, RFS.

(a) Cylinder (both internal and external). The candidate datum set for a cylinder is the set of axes of all actual mating envelopes of the datum feature. For secondary or tertiary datum features, the actual mating envelopes are constrained to be basically oriented (not located) to the higher precedence datums.

(b) Width (both internal and external). The candidate datum set for a width is the set of all center planes of all actual mating envelopes of the datum feature. For secondary or tertiary datum features, the actual mating envelopes are constrained to be basically oriented (not located) to the higher precedence datums.

Figure 4-2 shows an example of a tertiary datum at RFS oriented and not located to higher precedence datums. Datum plane *C* in the figure must be established from the datum feature under the constraint that the plane is parallel to datum *B* (and hence perpendicular to datum *A*). Datum plane *C* does not need to contain datum axis *B*.

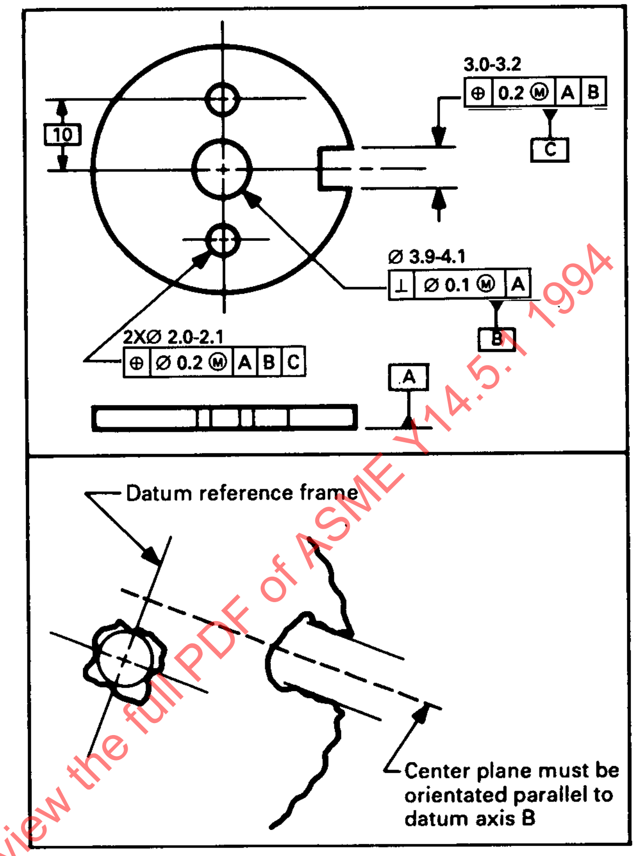


FIG. 4-2 TERTIARY DATUM IS BASICALLY ORIENTED ONLY

(c) Sphere (both internal and external). The candidate datum set for a sphere is the set of center points of all actual mating envelopes of the datum feature.

4.3.4 Datum Features Subject to Size Variation, MMC.

(a) Cylinder [External] {Internal}. The candidate datum set for a cylinder is the set of axes of all cylinders of MMC virtual condition size that [enclose] {are enclosed within} the datum feature. For secondary or tertiary datum features, the cylinders are constrained to be basically oriented and, as applicable, basically located to the higher precedence datums.

(b) Width [External] {Internal}. The candidate datum set for a width is the set of center planes of

all pairs of parallel planes separated by the MMC virtual condition size that [enclose] {are enclosed within} the datum feature. For secondary or tertiary datum features, the parallel planes are constrained to be basically oriented and, as applicable, basically located to the higher precedence datums.

(c) Sphere [External] {Internal}. The candidate datum set for a sphere is the set of center points of all spheres of MMC virtual condition size that [enclose] {are enclosed within} the datum feature. For secondary or tertiary datum features, the spheres are constrained as applicable to be basically located to the higher precedence datums.

4.3.5 Datum Features Subject to Size Variation, LMC.

(a) Cylinder [External] {Internal}. The candidate datum set for a cylinder is the set of axes of all cylinders of LMC virtual condition size that [are enclosed within] {enclose} the datum feature. For secondary or tertiary datum features, the cylinders are constrained to be basically oriented and, as applicable, basically located to the higher precedence datums.

(b) Width [External] {Internal}. The candidate datum set for a width is the set of center planes of all pairs of parallel planes separated by the LMC virtual condition size that [are enclosed within] {enclose} the datum feature. For secondary or tertiary datum features, the parallel planes are constrained to be basically oriented and, as applicable, basically located to the higher precedence datums.

(c) Sphere [External] {Internal}. The candidate datum set for a sphere is the set of center points of all spheres of LMC virtual condition size that [are enclosed within] {enclose} the datum feature. For secondary or tertiary datum features, the spheres are constrained as applicable to be basically located to the higher precedence datums.

4.4 ESTABLISHING DATUM REFERENCE FRAMES

The previous section establishes the rules for associating candidate datum sets with individual datum features. (While a candidate datum set is associated with an individual datum feature, datum precedence is used in the definition.) The candidate datums from these sets are used to construct candidate datum reference frames. The collection of datum reference frames that can be constructed in this way is called the candidate datum reference frame set. This Subsection establishes rules for constructing candidate datum reference frames and candidate datum reference frame sets.

The construction of a particular candidate datum reference frame proceeds as follows. A primary datum is selected from the candidate datum set associated with the primary datum feature. If a secondary datum is called out, the choice of primary datum establishes, by the rules of the previous subsection, a candidate datum set for the secondary datum feature. A secondary datum is chosen from this latter set. Similarly, if a tertiary datum is called out, the choice of primary and secondary datums establishes a candidate datum set for the tertiary datum feature. A tertiary datum is chosen from this last set. The choice of particular datums from the candidate datum sets determines a particular candidate datum reference frame, in a way described below. The set of candidate datum reference frames obtained by all possible choices of datums is the candidate datum reference frame set.

4.4.1 Degrees of Freedom.

The tables in Subsection 4.7 tabulate invariants for all possible datum reference frames. These tables show the possible combinations of datum geometries that can be used to establish datum reference frames. A datum reference frame may not fully constrain the coordinate system for locating and orienting tolerance zones. In other words, a datum reference frame may leave certain translational or rotational transformations free. An *invariant* in a datum reference frame is a quantity (distance or angle) that does not change under free transformations allowed by that frame.

The location of a completely defined datum reference frame is restricted in three translational directions (x, y, and z) and in three rotational orientations (u, v, and w) where u is rotation around the x axis, v is rotation around the y axis and w is rotation around the z axis. Tables 4-2 through 4-4 show the various combinations of datums and, for each case, the free transformations (degrees of freedom) and the invariants. Depending on how a designated feature is toleranced from a datum reference frame, a partially constrained datum reference frame may be sufficient to evaluate the designated feature. For example, if the datum reference frame is defined by only a plane (case 3.1 in Table 4-4), which leaves one rotational and two translational degrees of freedom, then it is sufficiently defined to evaluate the parallelism or perpendicularity of a designated feature with respect to the plane.

TABLE 4-1 SYMBOLS FOR DATUM TABLES

Symbol	Description
A	primary datum
B	secondary datum
C	tertiary datum
PT	point
AX	axis
PL	plane
{LI ...}	line through ...
{LI ... : ... }	line through ... such that ... is true
≠	not coincidental with
⊂	contained within
⊄	not contained within
//	parallel with
⊥	perpendicular to
∧	logical AND
∨	logical OR (one or the other, or both)
¬	logical NOT
∩	intersection
x, y, z	position in a cartesian coordinate system
u, v, w	rotation about x, y, z axis, respectively; yaw, pitch, roll, respectively
γ_z	angle relative to datum axis z
r	spherical radius: $\sqrt{x^2 + y^2 + z^2}$
P_z	cylindrical radius: $\sqrt{x^2 + y^2}$
—	no entry (e.g., not applicable, none)

4.4.2 Datum Precedence. The datum precedence (primary, secondary, and tertiary) in the feature control frame determines which datums arrest or constrain each of the degrees of freedom. The primary datum arrests three or more of the original six degrees of freedom. The secondary datum, if specified, arrests additional degrees of freedom that were not previously arrested by the primary datum. In some cases (for example, two orthogonal axes), two datums are sufficient to fully constrain the coordinate system. In such cases, a third datum cannot be meaningfully applied. Otherwise, the tertiary datum, if specified, arrests the balance of the degrees of freedom that were not previously arrested by the primary and secondary datums.

4.5 DATUM REFERENCE FRAMES FOR COMPOSITE TOLERENCES

In composite tolerancing, the feature-relating tolerances (lower tiers of the feature control frame) control only the orientation of the pattern. The candidate datum reference frame set for such a tolerance is the closure under translation of the candidate datum reference frame set established by the procedures given above.

4.6 MULTIPLE PATTERNS OF FEATURES

Where two or more patterns of features are related to common datum features referenced in the same

TABLE 4-2 POINT AS PRIMARY DATUM

Case	Datums A	Datums B	Datums C	Free xfrms	Invariants	Validity Conditions
1.1	PT	—	—	u, v, w	r	—
1.2	PT	PT	—	w	ρ_z, z, γ_z	A ≠ B
1.3	PT	PT	PT	—	all	(A ≠ B) ∧ (C ⊄ {LI A-B})
1.4	PT	PT	AX	—	all	(A ≠ B) ∧ (C ≠ {LI A-B})
1.5	PT	PT	PL	—	all	(A ≠ B) ∧ ¬ (C ⊥ {LI A-B})
1.6	PT	AX	—	—	all	A ⊄ B
1.7	PT	AX	—	w	ρ_z, z, γ_z	A ⊂ B
1.8	PT	AX	PT	—	all	(A ⊂ B) ∧ (C ⊄ B)
1.9	PT	AX	AX	—	all	(A ⊂ B) ∧ (B ≠ C)
1.10	PT	AX	PL	—	all	(A ⊂ B) ∧ ¬ (B ⊥ C)
1.11	PT	PL	—	w	ρ_z, z, γ_z	—
1.12	PT	PL	PT	—	all	C ⊄ {LI A: LI ⊥ B}
1.13	PT	PL	AX	—	all	C ≠ {LI A: LI ⊥ B}
1.14	PT	PL	PL	—	all	¬ (C // B)

TABLE 4-3 LINE AS PRIMARY DATUM

Case	Datums			Free xfrms	Invariants	Validity Conditions
	A	B	C			
2.1	AX	—	—	z, w	ρ_z, γ_z	—
2.2	AX	PT	—	—	all	$B \not\subset A$
2.3	AX	PT	—	w	ρ_z, z, γ_z	$B \subset A$
2.4	AX	PT	PT	—	all	$(B \subset A) \wedge (C \not\subset A)$
2.5	AX	PT	AX	—	all	$(B \subset A) \wedge (A \neq C)$
2.6	AX	PT	PL	—	all	$(B \subset A) \wedge \neg (A \perp C)$
2.7	AX	AX	—	—	all	$(A \neq B) \wedge \neg (A \mathbin{//} B)$
2.8	AX	AX	—	z	x, y, u, v, w	$(A \neq B) \wedge (A \mathbin{//} B)$
2.9	AX	AX	PT	—	all	$(A \neq B) \wedge (A \mathbin{//} B)$
2.10	AX	AX	AX	—	all	$(A \neq B) \wedge (A \mathbin{//} B) \wedge \neg (A \mathbin{//} C)$
2.11	AL	AX	PL	—	all	$(A \neq B) \wedge (A \mathbin{//} B) \wedge \neg (A \mathbin{//} C)$
2.12	AX	PL	—	—	all	$\neg ((A \mathbin{//} B) \vee (A \perp B))$
2.13	AX	PL	—	z	x, y, u, v, w	$A \mathbin{//} B$ (including $A \subset B$)
2.14	AX	PL	—	w	ρ_z, z, γ_z	$A \perp B$
2.15	AX	PL	PT	—	all	$(A \mathbin{//} B)$
2.16	AX	PL	PT	—	all	$(A \perp B) \wedge (C \not\subset A)$
2.17	AX	PL	AX	—	all	$(A \mathbin{//} B) \wedge \neg (A \mathbin{//} C)$
2.18	AX	PL	AX	—	all	$(A \perp B) \wedge (A \neq C)$
2.19	AX	PL	PL	—	all	$(A \mathbin{//} B) \wedge \neg (A \mathbin{//} C)$
2.20	AX	PL	PL	—	all	$(A \perp B) \wedge \neg (A \perp C)$

order of precedence and at the same material condition, as applicable, they are considered a composite pattern with the geometric tolerances applied simultaneously. If such interrelationship is not required, a notation such as SEP REQT *is placed adjacent to each applicable feature control frame.* When tolerances apply simultaneously, conformance and actual value of all features in a composite pattern must be evaluated with respect to a common datum reference frame selected from the candidate datum reference frame set. When a tolerance applies to each pattern as a separate requirement, conformance and actual value for each pattern can be evaluated using a different datum reference frame taken from the candidate datum reference frame set. All features within each pattern, however, must be evaluated with respect to a common datum reference frame. If composite tolerancing is used, the lower entries are always considered to be separate requirements for each pattern.

4.7 TABULATION OF DATUM SYSTEMS

This Subsection presents tables of datum systems. The first table presents the symbols used in the rest of the Section. The next three tables, organized by the geometry of the primary datum, present detailed information about each type of datum system. They list all valid combination of datum geometries, the free transformations remaining in the coordinate system associated with the datum reference frame, the invariant quantities under the free transformations, and the conditions under which the datum system is valid. The following example is from Table 4-3:

EXAMPLE FROM TABLE 4-2 LINE AS PRIMARY DATUM

Case	Datums			Free xfrms	Invariants	Validity Conditions
	A	B	C			
2.8	AX	AX	—	z	x, y, u, v, w	(A ≠ B) ∧ (A // B)

TABLE 4-4 PLANE AS PRIMARY DATUM

Case	Datums			Free xfrms	Invariants	Validity Conditions
	A	B	C			
3.1	PL	—	—	x, y, w	z, γ_z	—
3.2	PL	PT	—	w	z, γ_z, ρ_z	—
3.3	PL	PT	PT	—	all	C ⊄ {LI B : LI ⊥ A}
3.4	PL	PT	AX	—	all	C ≠ {LI B : LI ⊥ A}
3.5	PL	PT	PL	—	all	¬ (A // C)
3.6	PL	AX	—	—	all	¬ ((A // B) ∨ (A ⊥ B))
3.7	PL	AX	—	w	ρ_z, z, γ_z	A ⊥ B
3.8	PL	AX	—	x	y, z, u, v, w	A // B
3.9	PL	AX	PT	—	all	(A ⊥ B) ∧ (C ⊄ B)
3.10	PL	AX	PT	—	all	A // B
3.11	PL	AX	AX	—	all	(A ⊥ B) ∧ (B ≠ C)
3.12	PL	AX	AX	—	all	(A // B) ∧ ¬ (B // C)
3.13	PL	AX	PL	—	all	(A ⊥ B) ∧ ¬ (B ⊥ C)
3.14	PL	AX	PL	—	all	(A // B) ∧ ¬ (B // C)
3.15	PL	PL	—	x	y, z, u, v, w	¬ (A // B)
3.16	PL	PL	PT	—	all	¬ (A // B)
3.17	PL	PL	AX	—	all	¬ (A // B) ∧ ¬ (C // {LI (A ∩ B)})
3.18	PL	PL	PL	—	all	¬ (A // B) ∧ ¬ (C // {LI (A ∩ B)})

TABLE 4-5 GENERIC INVARIANT CASES

Index	Invariant Cases	Case Number(s)
1	r	1.1
2	P_z, γ_z	2.1
3	z, γ_z, P_z	1.2, 1.7, 1.11, 2.3, 2.14, 3.2, 3.7
4	z, γ_z	3.1
5	x, y, u, v, w *or* y, z, u, v, w	2.8, 2.13, 3.8, 3.15
6	all	all others

This shows case 2.8, a datum system consisting of a primary datum axis, a secondary datum axis, and no tertiary datum. The validity conditions indicate that this case applies only if the two axes (A and B) are parallel, but not equal. The only free transformation is translation along the z axis. As a result, the invariants include x and y coordinates, and all angle relationships between features and the datum reference frame.

Coordinate system labels are somewhat arbitrary. The following conventions apply. If the primary datum is a point, it establishes the origin. If the primary datum is a line, it establishes the z coordinate axis. If it is a plane, it establishes the x-y coordinate plane (and hence the direction of the z axis). Secondary and tertiary datums establish additional elements of the coordinate system.

For cases 1.7 through 1.10 in Table 4-2, if datum B is called out at RFS then datum axis B as derived from the actual datum feature will in general not pass through datum point A. In this case, for the purpose of establishing the datum reference frame, datum axis B should be translated so as to pass through datum point A.

For cases 2.3 through 2.6 in Table 4-3, if datum B is called out at RFS then datum point B as derived from the actual datum feature will in general not lie on datum axis A. In this case, for the purpose of establishing the datum reference frame, datum B should be projected onto datum axis A.

Table 4-5 summarizes the contents of Tables 4-2 through 4-4 according to the free transformations and invariants. It can be seen that, subject to renaming of directions in the coordinate systems, there are only six distinct cases of datum systems. Table 4-5 cross-references the distinct cases to the cases in the previous tables.

The symbols used in Tables 4-2 through 4-5 are presented in Table 4-1.

5 TOLERANCES OF LOCATION

5.1 GENERAL

This Section establishes the principles of tolerances of location; included are position, concentricity, and symmetry used to control the following relationships:

(a) center distance between [features of size, such as] holes, slots, bosses, and tabs;

(b) location of features (such as in (a) above) as a group with respect to datum reference frames;

(c) coaxiality of features;

(d) concentricity or symmetry of features — center distances of correspondingly-located feature elements equally disposed about a datum axis or plane.

5.1.1 Material Condition Basis. *Positional tolerances are applied on an MMC, RFS, or LMC basis.* A position tolerance may be explained either in terms of the surface of the actual feature or in terms of size and the resolved geometry (center point, axis, or center plane) of the applicable (mating or minimum material) actual envelope. These two interpretations will be called the *surface interpretation* and the *resolved geometry interpretation*, respectively. (See Subsection 5.2 and the following for the precise definitions of positional tolerancing interpretations.) For MMC and LMC callouts, these explanations are not equivalent. They differ in part because the resolved geometry interpretation relies on an assumption that the feature is of perfect form and in part because the derivation of the surface interpretation assumes perfect orientation.

Two examples will illustrate the issues. Consider the illustration in Fig. 5-1. Assume that the hole shown in the figure is controlled by a zero position tolerance at MMC. The MMC virtual condition boundary has a diameter equal to the MMC diameter of the hole. The actual hole was manufactured with poor form, but assume that it is within the limits of size. (See para. 2.3.1.) The hole does not violate the virtual condition boundary, and, as explained below, would be accepted according to the surface interpretation. Using the resolved geometry interpretation, however, the hole is apparently further away from true position than allowed by the combined effects of the position tolerance (zero) and the bonus tolerance resulting from the actual mating size of the hole. (The virtual condition boundary extends beyond the actual mating envelope.) The hole would not be accepted according to the resolved geometry interpretation.

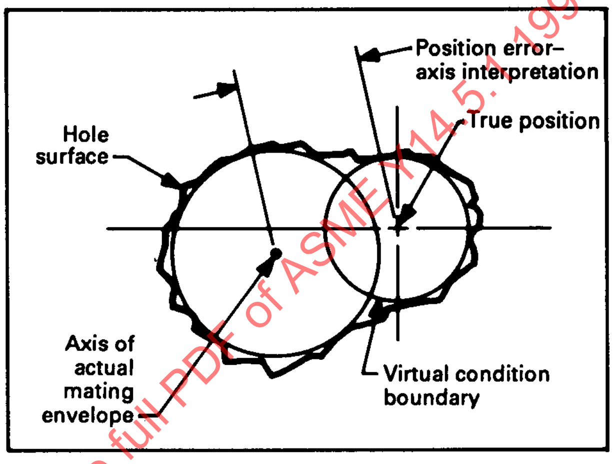


FIG. 5-1 ILLUSTRATION OF DIFFERENCE BETWEEN SURFACE AND SIZE INTERPRETATIONS OF POSITION TOLERANCING FOR A CYLINDRICAL HOLE

Figure 5-2 shows the converse situation. Assume that the shaft shown in the figure is controlled by a position tolerance t at MMC. Assume also that the shaft was manufactured with perfect form. (This assumption is not necessary, but simplifies the example.) If the radius of the shaft is r_{AM} and the MMC radius is r_{MMC}, the radius of the tolerance zone for the axis is $r_{MMC} - r_{AM} + t/2$. If the height of the shaft is h, and the axis of the actual shaft is tilted to an extreme orientation within the tolerance zone, a simple geometric analysis shows that point $\vec{P}$ lies outside the virtual condition boundary by a distance

$$r_{AM}[\sqrt{1+[(t+2(r_{MMC}-r_{AM}))/h]^2}-1].$$

Thus, a feature may be acceptable according to a resolved geometry interpretation but fail according to the surface interpretation.

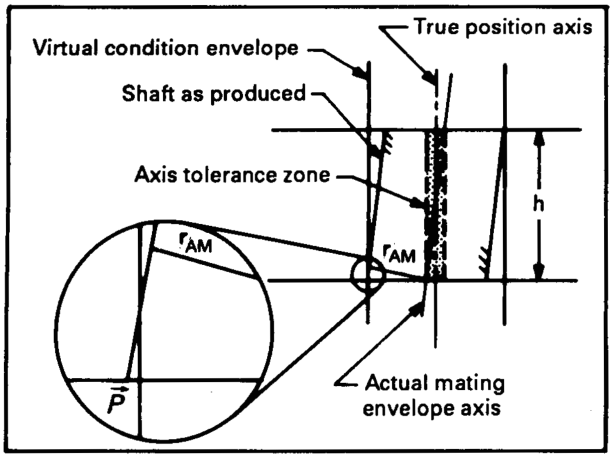


FIG. 5-2 THE SHAFT SATISFIES THE RESOLVED GEOMETRY INTERPRETATION BUT VIOLATES THE VIRTUAL CONDITION BOUNDARY

Throughout most of this Section, both a surface interpretation and a resolved geometry interpretation are supplied. In a few cases (e.g., projected tolerance zones) only a surface interpretation is provided. Whenever the two interpretations do not produce equivalent results, the surface interpretation shall take precedence.

5.1.2 Patterns of Features. For the purposes of this Standard all tolerances of location are considered to apply to patterns of features, where a pattern may consist of only a single feature. The control of the location of the pattern as a group is called the *pattern-locating tolerance zone framework (PLTZF)*. When the pattern consists of two or more features, there is the possibility, through the use of composite tolerancing, to control the relative location of features within the pattern. This is done by specifying a secondary location tolerance, called the *feature-relating tolerance zone framework (FRTZF)*, in conjunction with the PLTZF. There may be more than one FRTZF for a pattern. All features within a single pattern are controlled simultaneously. That is, all features must be evaluated with respect to a single datum reference frame from the candidate datum reference frame set for the control.

5.2 POSITIONAL TOLERANCING

This Subsection presents a general explanation of positional tolerancing for features of size. A positional tolerance can be explained in terms of a zone within which the resolved geometry (center point, axis, or center plane) of a feature of size is permitted to vary from true (theoretically exact) position. Basic dimensions establish the true position from specified datum features and between interrelated features. (Note: some position tolerance specifications can be explained in terms of a surface boundary.)

Throughout this Subsection, whenever the true position is understood from context, the notation $r(\vec{P})$ will denote the distance from a point $\vec{P}$ to the true position. For spheres, $r(\vec{P})$ is the distance to the true position center point. For cylinders, $r(\vec{P})$ is the distance between $\vec{P}$ and the true position axis. For parallel plane features, $r(\vec{P})$ is the distance between $\vec{P}$ and the true position center plane. These definitions should also be understood to be for a particular choice of datum reference frame from the candidate datum reference frame set. Throughout this Section, all spherical and cylindrical sizes are in terms of radius unless otherwise specified. All tolerance values are assumed to be diameters for spheres and cylinders, and full widths for parallel planes, in accord with common practice.

TABLE 5-1 DEFINITION OF POSITION TOLERANCE ZONE, SURFACE INTERPRETATION

		Material Condition Basis	
		MMC or RFS	LMC
Feature Type	Internal	$r(\vec{P}) < b$	$r(\vec{P}) > b$
	External	$r(\vec{P}) > b$	$r(\vec{P}) < b$

5.2.1 In Terms of the Surface of a Feature.

(a) Definition. For a pattern of features of size, a position tolerance specifies that the surface of each actual feature must not violate the boundary of a corresponding position tolerance zone. Each boundary is a sphere, cylinder, or pair of parallel planes of size equal to the collective effect of the limits of size, material condition basis, and applicable positional tolerance. Each boundary is located and oriented by the basic dimensions of the pattern. Each position tolerance zone is a volume defined by all points $\vec{P}$ that satisfy the appropriate equation from Table 5-1, where b is a position tolerance zone size parameter (radius or half-width).

Figure 5-3 illustrates the tolerance zone for a cylindrical hole at MMC or RFS, or a shaft at LMC.

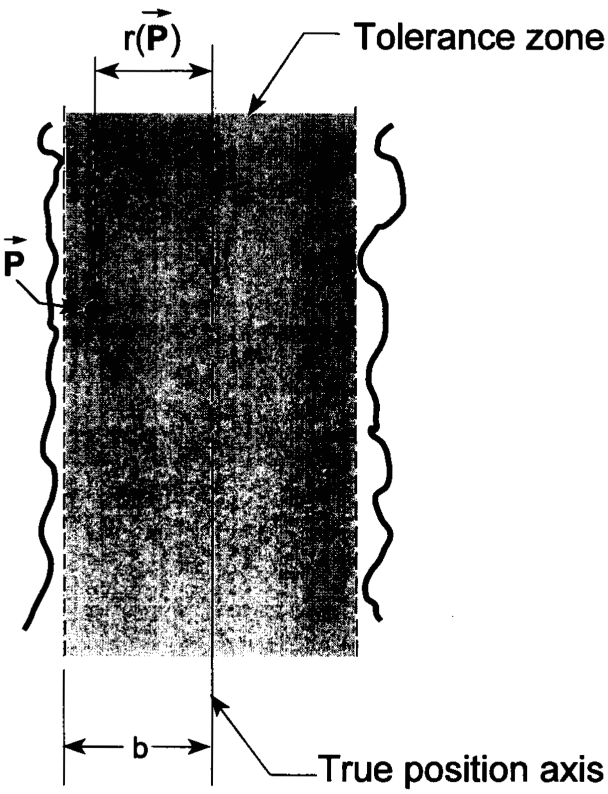


FIG. 5-3 TOLERANCE ZONE AND CONFORMANCE: HOLES AT MMC OR RFS; SHAFTS AT LMC

TABLE 5-2 SIZE OF POSITION TOLERANCE ZONE, SURFACE INTERPRETATION

	b	Material Condition Basis: MMC	RFS	LMC
Feature Type	Internal	$r_{MMC} - \frac{t_0}{2}$	$r_{AM} - \frac{t_0}{2}$	$r_{LMC} + \frac{t_0}{2}$
	External	$r_{MMC} + \frac{t_0}{2}$	$r_{AM} + \frac{t_0}{2}$	$r_{LMC} - \frac{t_0}{2}$

The tolerance zone is a cylindrical volume. Figure 5-4 illustrates the tolerance zone for a tab at MMC or RFS, or a slot at LMC. The tolerance zone is two disjoint half-spaces bounded by parallel planes.

(b) Conformance. A feature conforms to a position tolerance t_0 at a specified material condition basis if all points of the feature lie outside some posi-

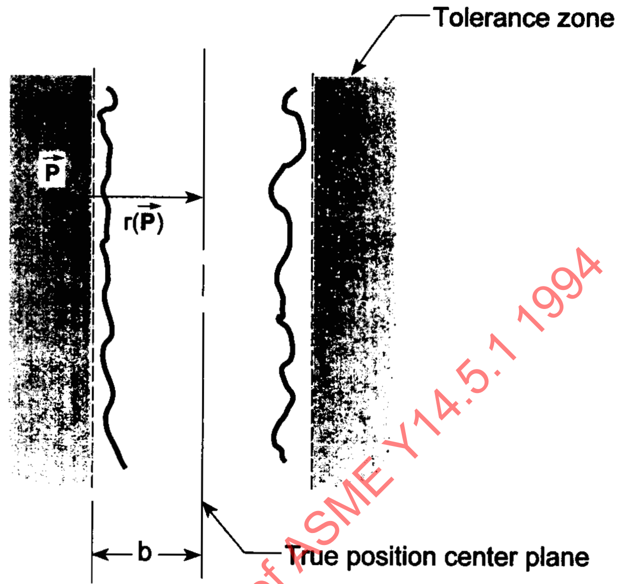


FIG. 5-4 TOLERANCE ZONE AND CONFORMANCE: TABS AT MMC OR RFS; SLOTS AT LMC

tion zone as defined above with b determined by the appropriate value from Table 5-2.

The surface must conform to the applicable size limits. In the case of an internal feature (spherical hollow, hole, or slot), there is a further condition that the feature must surround the tolerance zone.

For MMC or LMC material condition basis, the boundary defined by $r(\vec{P}) = b$, with b as given here, is called the virtual condition.

(c) Actual value. The actual value of position deviation is the smallest value of t_0 to which the feature conforms.[1]

5.2.2 In Terms of the Resolved Geometry of a Feature.

(a) Definition For features within a pattern, a position tolerance specifies that the resolved geometry (center point, axis, or center plane, as applicable) of each actual mating envelope (for features at MMC or RFS) or actual minimum material envelope (for features at LMC) must lie within a corresponding positional tolerance zone. Each zone is bounded by a sphere, cylinder, or pair of parallel planes of size equal to the total allowable tolerance for the corres-

[1]For LMC and MMC controls the actual value of deviation can be negative. A negative actual value can be interpreted as the unused portion of the bonus tolerance resulting from the departure of the feature from the applicable limit of size.

TABLE 5-3 SIZE OF POSITION TOLERANCE ZONE, RESOLVED GEOMETRY INTERPRETATION

Material Condition Basis

Feature Type	*b*	MMC	RFS	LMC
Feature Type	Internal	$\frac{t_0}{2} + (r_{AM} - r_{MMC})$	$\frac{t_0}{2}$	$\frac{t_0}{2} + (r_{LMC} - r_{AMM})$
	External	$\frac{t_0}{2} + (r_{MMC} - r_{AM})$	$\frac{t_0}{2}$	$\frac{t_0}{2} + (r_{AMM} - r_{LMC})$

ponding feature. Each zone is located and oriented by the basic dimensions of the pattern. A position tolerance zone is a spherical, cylindrical, or parallel-plane volume defined by all points $\vec{P}$ that satisfy the equation $r(\vec{P}) \leq b$, where b is the radius or half-width of the tolerance zone.

Figure 5-5 illustrates the definition for holes at MMC and RFS and for shafts at LMC. The figure shows the position of a point on the axis of the actual envelope that is outside the tolerance zone. A similar figure for holes at LMC or shafts at MMC or RFS would show the actual envelope surrounding the feature surface. The feature axis extends for the full length of the feature.

(b) Conformance. A feature conforms to a position tolerance t_0 at a specified material condition basis if all points of the resolved geometry of the applicable envelope (as determined by the material condition basis) lie within some position zone as defined above with b determined by the appropriate formula from Table 5-3. Furthermore, the surface must conform to the applicable size limits.

(c) Actual value. The position deviation of a feature is the diameter of the smallest tolerance zone (smallest value of b) which contains the center point or all points on the axis or center plane (within the extent of the feature) of the applicable actual envelope of the feature.

5.3 PROJECTED TOLERANCE ZONE

(a) Definition. For a cylindrical or parallel-plane feature, a projected tolerance specifies that a volume, called a verifying volume, with a boundary of perfect form, called a verifying boundary, can be defined such that the following two conditions hold. First, the axis or center plane of the verifying boundary is contained within a projected position tolerance zone,

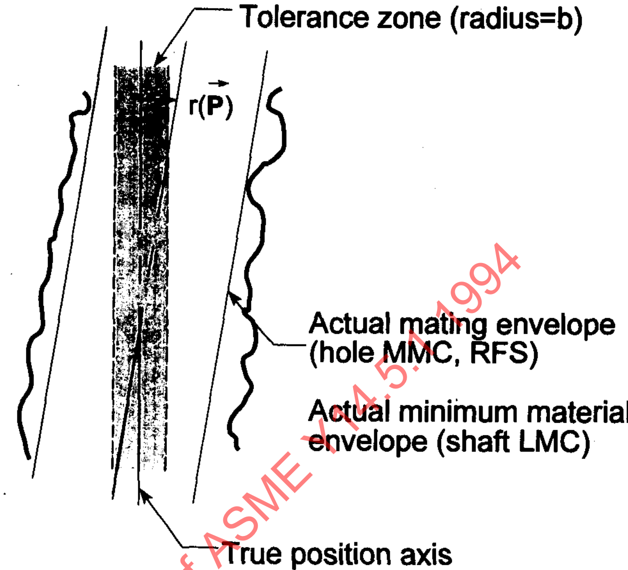


FIG. 5-5 TOLERANCE ZONE AND CONFORMANCE: HOLES AT MMC OR RFS; SHAFTS AT LMC

TABLE 5-4 DEFINITION OF VERIFYING VOLUME FOR PROJECTED TOLERANCE ZONE

Material Condition Basis

Feature Type		MMC or RFS	LMC
Feature Type	Internal	$r(\vec{P}) < w$	$r(\vec{P}) > w$
	External	$r(\vec{P}) > w$	$r(\vec{P}) < w$

itself a boundary of perfect form extending outward from the feature by the specified projection length. Second, the surface of the feature does not violate the verifying volume.

A projected position tolerance zone is a cylindrical or parallel-plane volume defined by all points $\vec{P}$ that satisfy the equation $r(\vec{P}) \leq b$, where b is the radius or half-width of the tolerance zone. A verifying volume is a cylindrical or parallel-plane volume defined by all points $\vec{P}$ that satisfy the appropriate equation from Table 5-4, where w is a size parameter for the verifying volume.

Figure 5-6 illustrates a typical case. The projected tolerance zone is positioned and oriented by the choice of datum reference frame. A plane perpendic-

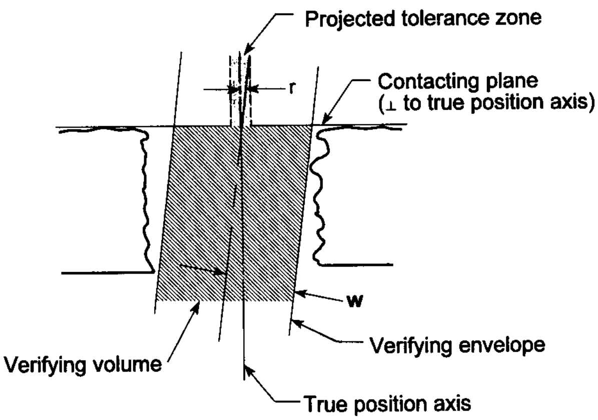


FIG. 5-6 PROJECTED TOLERANCE ZONE FOR A HOLE

ular to the true position axis is located to contact the part surface that defines the end of the cylindrical feature. The height of the zone is the specified projection length and starts at the point where the true position axis intersects the contacting plane. The verifying volume is shown for a hole at MMC or RFS. (A similar picture would apply for a shaft at LMC. For a shaft at MMC or RFS, or a hole at LMC, the verifying envelope would surround the feature.)

(b) Conformance. A feature conforms to a position tolerance t_0, projected a distance h, and at a specified material condition basis, if there exists at least one verifying volume for which the following conditions hold. All points of the feature lie outside the verifying volume as defined above with w determined from the material basis as follows: for MMC, $w = r_{MMC}$; for RFS, $w = r_{AM}$; and for LMC, $w = r_{LMC}$. The verifying envelope satisfies $r(\vec{P}) \le t_0/2$ for all points $\vec{P}$ on the resolved geometry starting at the intersection of the resolved geometry with the contacting plane and ending at the intersection of the resolved geometry with a second plane parallel to the contacting plane and separated from it by a distance h.

Note that for RFS features, this definition can also be considered the resolved geometry interpretation. No resolved geometry interpretation is provided for MMC or LMC tolerances.

(c) Actual value. The position deviation of a feature is the size of the smallest projected tolerance zone such that the resolved geometry of the actual mating envelope lies within the tolerance zone for the full projection height.

TABLE 5-5 DEFINITION OF CONICAL TOLERANCE ZONE, SURFACE INTERPRETATION

		Material Condition Basis	
		MMC or RFS	LMC
Feature Type	Internal	$r(\vec{P}) < b(\vec{P})$	$r(\vec{P}) > b(\vec{P})$
	External	$r(\vec{P}) > b(\vec{P})$	$r(\vec{P}) < b(\vec{P})$

5.4 CONICAL TOLERANCE ZONE

A conical position tolerance zone is specified by different position tolerance values at each end of a cylindrical feature. A conical tolerance can be interpreted either in terms of the surface of the feature or in terms of the axis of the feature.

5.4.1 In Terms of the Surface of the Feature.

(a) Definition. For a pattern of cylindrical features, a position control tighter at one end of the features than the other specifies that the surface of each actual feature must not violate a corresponding perfect form conical boundary. This boundary is a frustum of height and diameters equal to the collective effects of the limits of size, material condition basis, and applicable positional tolerances at each end of the feature. The boundary is located and oriented by the basic dimensions of the feature. A position tolerance zone is a conical volume defined by all points $\vec{P}$ that satisfy the appropriate equation from Table 5-5, where $b(\vec{P})$ is the radius of the tolerance zone at height $\vec{P}$. The radius $b(\vec{P})$ is related to the position tolerance zone size parameters r_1 and r_2 by:

$$b(\vec{P}) = r_1\,(1 - \gamma(\vec{P})) + r_2\gamma(\vec{P})$$

where

$$\gamma(\vec{P}) = \frac{(\vec{P} - \vec{P}_1) \cdot (\vec{P}_2 - \vec{P}_1)}{|\vec{P}_2 - \vec{P}_1|^2}$$

is the position of $\vec{P}$ along the axis between $\vec{P}_1$ and $\vec{P}_2$, scaled so that $\gamma(\vec{P}_1) = 0$ and $\gamma(\vec{P}_2) = 1$.

Figure 5-7 illustrates the definition for holes at MMC and RFS. (Shafts would have a similar picture.) A similar figure for holes at LMC or shafts at MMC or RFS would show the envelope surrounding

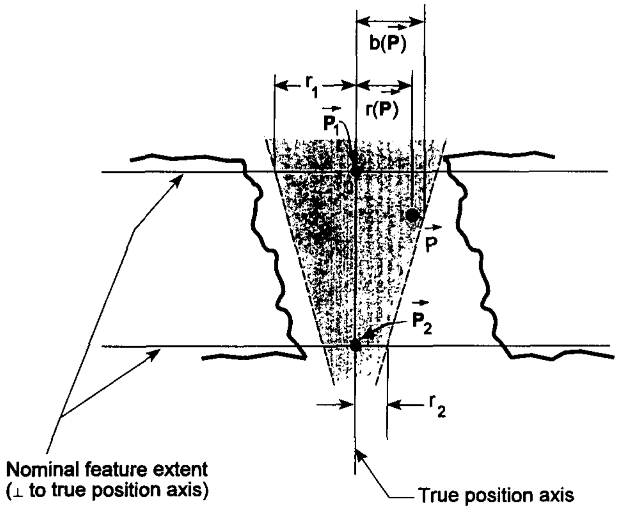


FIG. 5-7 SURFACE INTERPRETATION OF CONICAL TOLERANCE ZONE FOR HOLES AT MMC OR RFS

the feature surface. The tolerance zone axis extends between points $\vec{P}_1$ and $\vec{P}_2$, which are the intersection of the true position axis with two planes, one at each end of the basic feature, at nominal distance and nominally located and oriented relative to the datum reference frame.

(b) Conformance. A cylindrical feature conforms to position tolerances t_1 and t_2 at a specified material condition basis if all points of the feature lie outside some position zone as defined above, with $r_i (i = 1, 1)$ determined according to Table 5-6. The surface must also conform to the applicable size limits. In the case of a hole, there is a further condition that the hole must surround the tolerance zone.

(c) Actual value. No definition for actual value of position deviation is provided for the surface interpretation. Refer below to the axis interpretation for a definition of actual value.

5.4.2 In Terms of the Axis of the Feature.

(a) Definition. For the axes of cylindrical features within a pattern, a position tolerance tighter at one end specifies that the axes of the actual mating envelopes (for features at MMC or RFS) or of the actual minimum material envelopes (for features at LMC) must lie within corresponding positional tolerance zones. Each of these zones is bounded by a frustum of height and diameters equal to the collective effects of the limits of size, material condition basis, and applicable positional tolerances at each end of the feature. The axis of the frustum is located and oriented by the basic dimensions of the feature. The frustum is located along the axis by the nominal surfaces bounding the feature. The position tolerance zone is a conical volume defined by all points $\vec{P}$ that satisfy the equation $r(\vec{P}) \le b(\vec{P})$, where $b(\vec{P})$ is the radius of the tolerance zone at the height along the axis of $\vec{P}$. (See the surface interpretation, para. 5.4.1, for details.)

Figure 5-8 illustrates the axis definition for holes. The tolerance zone axis extends between points $\vec{P}_1$

TABLE 5-6 SIZES OF CONICAL TOLERANCE ZONE, SURFACE INTERPRETATION

		Material Condition Basis		
	r_1	MMC	RFS	LMC
Feature Type	Internal	$r_{MMC} - \frac{t_i}{2}$	$r_{AM} - \frac{t_i}{2}$	$r_{LMC} + \frac{t_i}{2}$
	External	$r_{MMC} + \frac{t_i}{2}$	$r_{AM} + \frac{t_i}{2}$	$r_{LMC} - \frac{t_i}{2}$

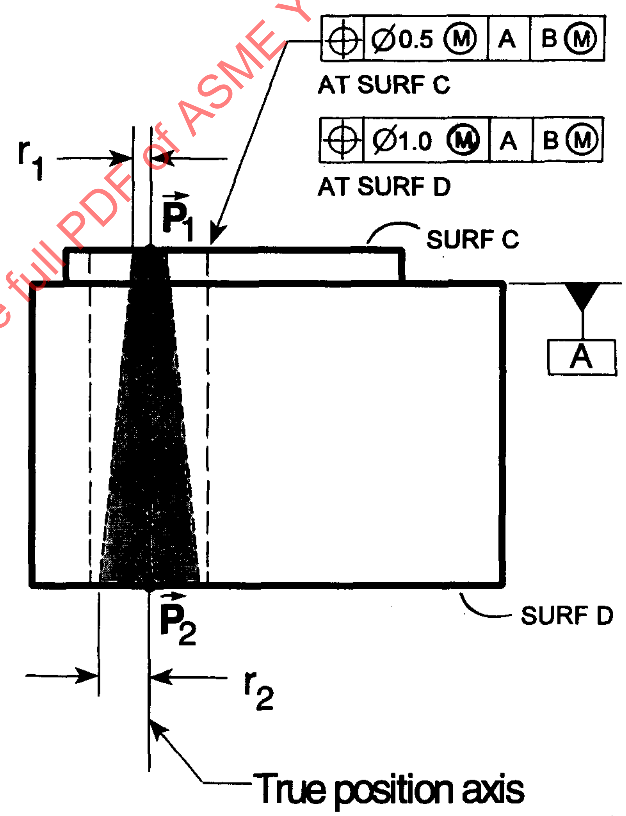


FIG. 5-8 AXIS INTERPRETATION OF CONICAL TOLERANCE ZONE FOR HOLES AT MMC OR RFS

TABLE 5-7 SIZES OF CONICAL TOLERANCE ZONE, AXIS INTERPRETATION

	r_i	Material Condition Basis: MMC	RFS	LMC
Feature Type	Internal	$\frac{t_i}{2} + (r_{AM} - r_{MMC})$	$\frac{t_i}{2}$	$\frac{t_i}{2} + (r_{LMC} - r_{AMM})$
	External	$\frac{t_i}{2} + (r_{MMC} - r_{AM})$	$\frac{t_i}{2}$	$\frac{t_i}{2} + (r_{AMM} - r_{LMC})$

and $\vec{P}_2$, which are the intersection of the true position axis with the nominal surfaces bounding the feature.

(b) Conformance. A cylindrical feature conforms to position tolerances t_1 and t_2 at a specified material condition basis if all points on the axis of the applicable envelope (as determined by the material condition basis) lie within some position zone as defined above, with r_1 (i = 1, 2) determined by the appropriate formula from Table 5-7. Furthermore, the surface must conform to the applicable size limits.

(c) Actual value. A cylindrical feature controlled by a conical tolerance zone has two actual values for position deviation, one at each end of the feature. The actual value at each end is the smallest diameter circle that contains the axis of the actual mating envelope at that end. Each circle is in the plane perpendicular to the true position axis at the end point of the feature axis, and is centered on the true position axis. In the case that the actual mating envelope can rock, it may be possible to decrease the actual value of position deviation at one end at the expense of the deviation at the other end. No rule is defined for selecting among possible pairs of actual values.

5.5 BIDERECTIONAL POSITIONAL TOLERANCING

A bidirectional positional tolerance zone for a cylindrical feature is specified by different position tolerance values in different directions perpendicular to the basic feature axis. Bidirectional positional tolerancing results in two distinct tolerance zones for locating each cylindrical feature. Each tolerance zone is considered separately in the following. As with other tolerances, however, rules for simultaneous or separate requirements apply to the components of a bidirectional positional tolerance. (See Subsection 4.6.) Bidirectional positional tolerancing may be applied in either a rectangular or a polar (cylindrical) coordinate system. Rectangular bidirectional tolerancing can be explained in terms of either the surface or the axis of the feature. An axis interpretation only is provided for polar bidirectional tolerancing.

TABLE 5-8 DEFINITION OF BIDIRECTIONAL POSITION TOLERANCE ZONE, SURFACE INTERPRETATION

		Material Condition Basis: MMC or RFS	LMC
Feature Type	Internal	$r(\vec{P}) < b$	$r(\vec{P}) > b$
	External	$r(\vec{P}) > b$	$r(\vec{P}) < b$

GENERAL NOTE: In this table, $r(\vec{P})$ is the distance from $\vec{P}$ to the resolved geometry of the tolerance zone boundary. The tolerance zone boundary is a cylinder for a hole at MMC or RFS and for shafts at LMC; it is a pair of parallel planes for shafts at MMC or RFS and for holes at LMC.

5.5.1 In Terms of the Surface of the Feature. This Section establishes the surface interpretation of bidirectional positional tolerancing when applied in a rectangular coordinate system.

(a) Definition. For a pattern of cylindrical features, each bidirectional positional tolerance specifies that each surface must not violate a tolerance boundary. For holes at MMC or RFS and shafts at LMC, each tolerance boundary is a cylinder of diameter equal to the collective effects of the limits of size, material condition basis, and applicable position tolerance. Each boundary is located and oriented, by the basic dimensions of the pattern and by the applicable direction of tolerance control, such that the axis of each boundary lies in the plane containing the true position axis of the corresponding feature and normal to the direction in which the tolerance applies. The orientation and position of the boundary axis within this plane is unconstrained.

For holes at LMC and shafts at MMC or RFS, each tolerance boundary is a pair of parallel planes separated by a distance equal to the collective effects of the limits of size, material condition basis, and applicable position tolerance. The center plane of each boundary is that plane containing the axis of the corresponding feature and normal to the direction in which the tolerance applies.

A positional tolerance zone is a volume defined by all points $\vec{P}$ that satisfy the appropriate equation from Table 5-8, where b is a position tolerance zone size parameter (radius or half-width).

(b) Conformance. A cylindrical feature conforms to a bidirectional positional tolerance t_0 at a specified material condition basis if all points of the feature lie outside some position tolerance as defined above with b determined by the appropriate value from Table 5-9.

Figure 5-9 shows an example of bidirectional tolerancing of a hole at MMC. Each callout creates its own cylindrical position tolerance zone. The zone corresponding to the 0.4 mm tolerance, shown in the bottom left, is free to be positioned and oriented only in the plane indicated by the vertical dashed line. Similarly, the zone corresponding to the 0.2 mm tolerance, shown in the bottom right, can only move and tilt left-to-right in the view shown. Each of these planes of motion are determined by the basic dimensions from the indicated datums.

A similar example is illustrated for shafts at MMC in Fig. 5-10. In this case, each callout creates a toler-

TABLE 5-9 SIZE OF BIDIRECTIONAL POSITION TOLERANCE ZONE, SURFACE INTERPRETATION

		Material Condition Basis		
	b	MMC	RFS	LMC
Feature Type	Internal	$r_{MMC} - \frac{t_0}{2}$	$r_{AM} - \frac{t_0}{2}$	$r_{LMC} + \frac{t_0}{2}$
	External	$r_{MMC} + \frac{t_0}{2}$	$r_{AM} + \frac{t_0}{2}$	$r_{LMC} - \frac{t_0}{2}$

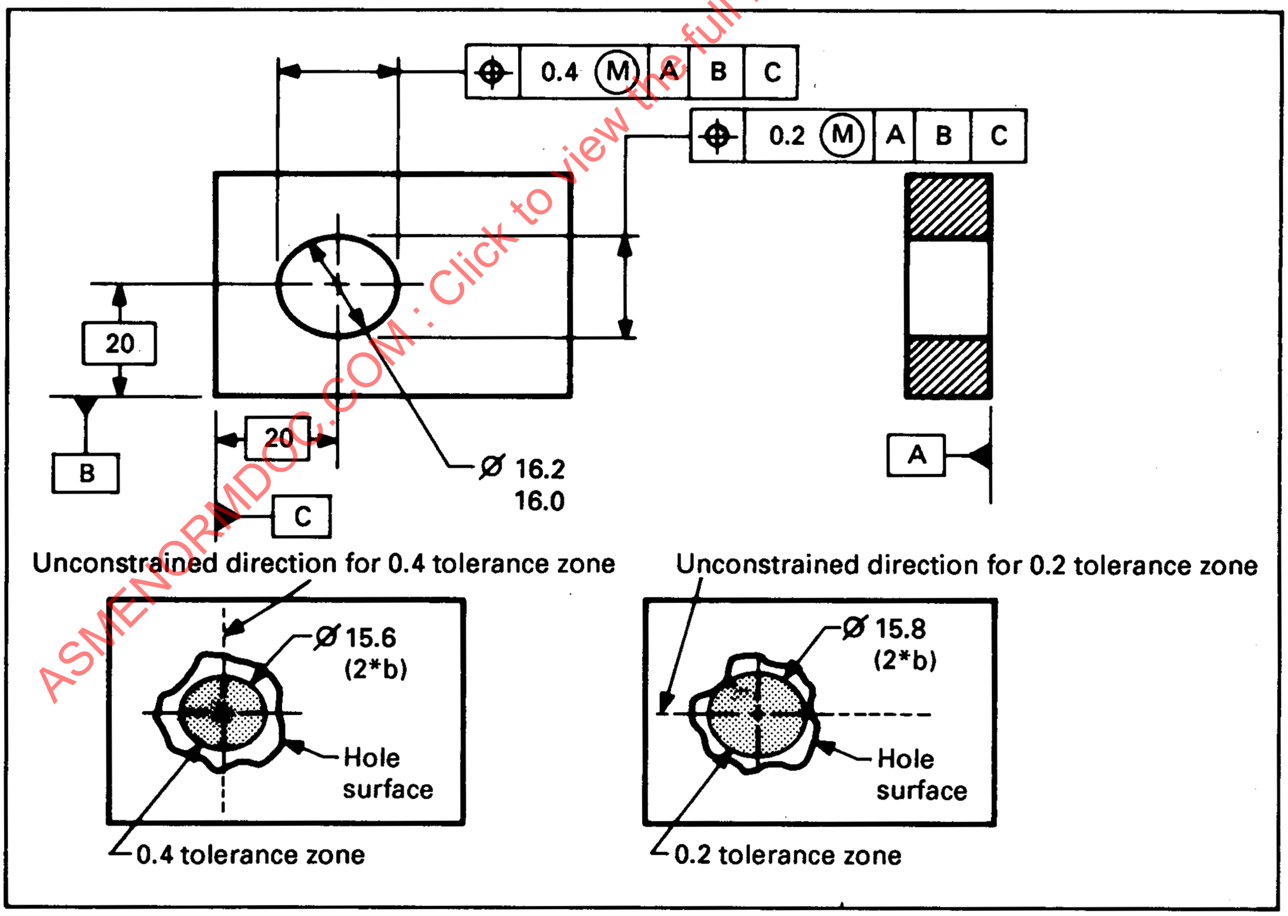


FIG. 5-9 BIDIRECTIONAL HOLE TOLERANCES AT MMC. THE AXIS OF EACH TOLERANCE BOUNDARY IS CONSTRAINED TO LIE IN THE INDICATED PLANCE

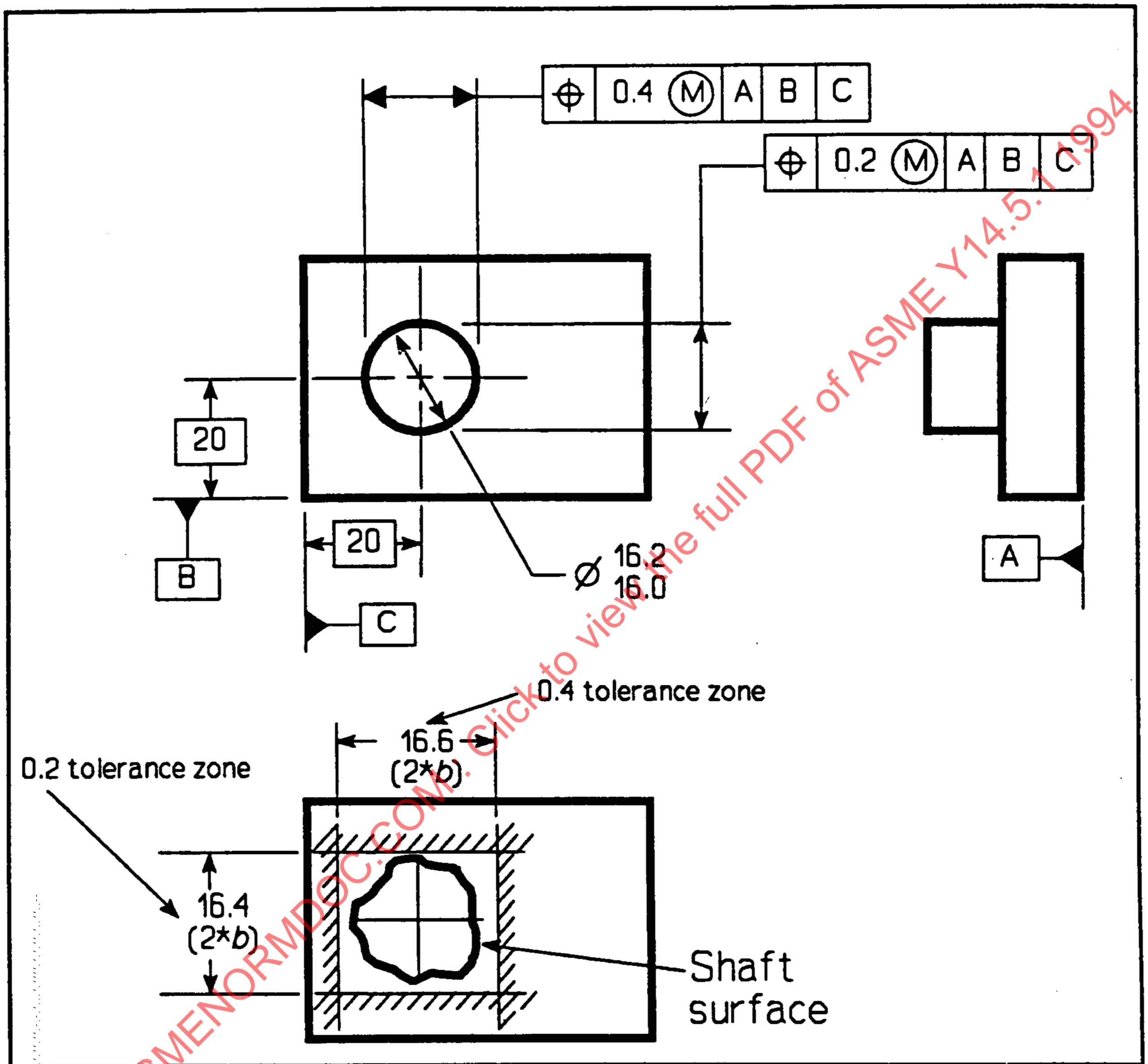


FIG. 5-10 ILLUSTRATION OF BIDIRECTIONAL POSITION TOLERANCING OF A SHAFT AT MMC. THE TOLERANCE ZONE FOR EACH CALLOUT IS BOUNDED BY A PAIR OF PARALLEL PLANES

TABLE 5-10 SIZE OF BIDIRECTIONAL POSITION TOLERANCE ZONE, AXIS INTERPRETATION

		Material Condition Basis		
	b	MMC	RFS	LMC
Feature Type	Internal	$\frac{t_0}{2} + (r_{AM} - r_{MMC})$	$\frac{t_0}{2}$	$\frac{t_0}{2} + (r_{LMC} - r_{AMM})$
	External	$\frac{t_0}{2} + (r_{MMC} - r_{AM})$	$\frac{t_0}{2}$	$\frac{t_0}{2} + (r_{AMM} - r_{LMC})$

ance zone bounded by parallel planes. The zone corresponding to the 0.4 mm tolerance is bounded by the vertical planes separated by 16.6 mm. The zone corresponding to the 0.2 mm tolerance is bounded by the horizontal planes separated by 16.4 mm.

(c) Actual value. No definition for actual value of bidirectional position deviation is provided in terms of the surface of the feature. Refer below to the axis interpretation for a definition of actual value.

5.5.2 In Terms of the Axis of the Feature. This Section establishes the axis (resolved geometry) interpretation of bidirectional positional tolerancing when applied in a rectangular coordinate system.

(a) Definition. For axes of cylindrical features within a pattern, bidirectional position tolerances specify that the axis of each actual mating envelope (for features at MMC or RFS) or minimum material envelope (for features at LMC) must lie within two corresponding positional tolerance zones. Each zone is bounded by two parallel planes separated by a distance equal to the total allowable tolerance for the corresponding feature, including any effects of feature size. Each zone is located and oriented by the basic dimensions of the pattern. A bidirectional position tolerance zone is a (slab) volume defined by all points $\vec{P}$ that satisfy the equation $r(\vec{P}) \leq b$, where b is half the thickness of the tolerance zone.

(b) Conformance. A cylindrical feature conforms to a position tolerance t_0 at a specified material condition basis if all points on the axis of the applicable envelope (as determined by the material condition basis) lie within some position zone as defined above with b determined by the appropriate formula from Table 5-10. Furthermore, the surface must conform to the applicable size limits.

(c) Actual value. The position deviation of a feature is the thickness of the smallest tolerance zone to which the axis conforms. (Note: a feature will usually be controlled by more than one directional tolerance; there is a distinct actual value for each tolerance callout.)

5.5.3 Polar Bidirectional Tolerancing in Terms of the Axis of the Feature. This Section establishes the axis (resolved geometry) interpretation of bidirectional positional tolerancing when applied in a cylindrical coordinate system. (While the term "polar" is used in ASME Y14.5M-1994, and used herein for consistency, a cylindrical coordinate system is actually being used. The tolerances are specified in the plane normal to the axis of the cylindrical coordinate system.)

(a) Definition. For axes of cylindrical features within a pattern, polar bidirectional position tolerances specify that the axes of the actual mating envelopes (for features at MMC or RFS) or of the minimum material envelopes (for features at LMC) must lie within corresponding positional tolerance zones. Each zone is bounded radially by two concentric cylindrical arcs and tangentially by two planes symmetrically disposed about the true position of the feature and oriented at the basic polar angle of the feature. The plane separation and the difference in cylindrical arc radii are each equal to the total allowable tolerance for the corresponding feature, including any effects of feature size. Each zone is located and oriented by the basic dimensions of the pattern. A polar bidirectional position tolerance zone is a (cylindrical shell) volume defined by all points $\vec{P}$ that satisfy the two equations:

$$|\rho_{\vec{P}} - \rho_0| < b_r$$

and

$$|(\vec{P} - \vec{A}) \cdot \hat{N}_t| \leq b_t$$

where

$\vec{A}$ = a point on the true position axis of the feature

$\rho_{\vec{P}}$ = the distance of $\vec{P}$ from the axis of the polar (cylindrical) coordinate system

ρ_o = the distance of the true position axis from the axis of the polar (cylindrical) coordinate system

$\hat{N}_t$ = the direction vector of the plane containing the axis of the polar (cylindrical) coordinate system and the true position axis of the feature

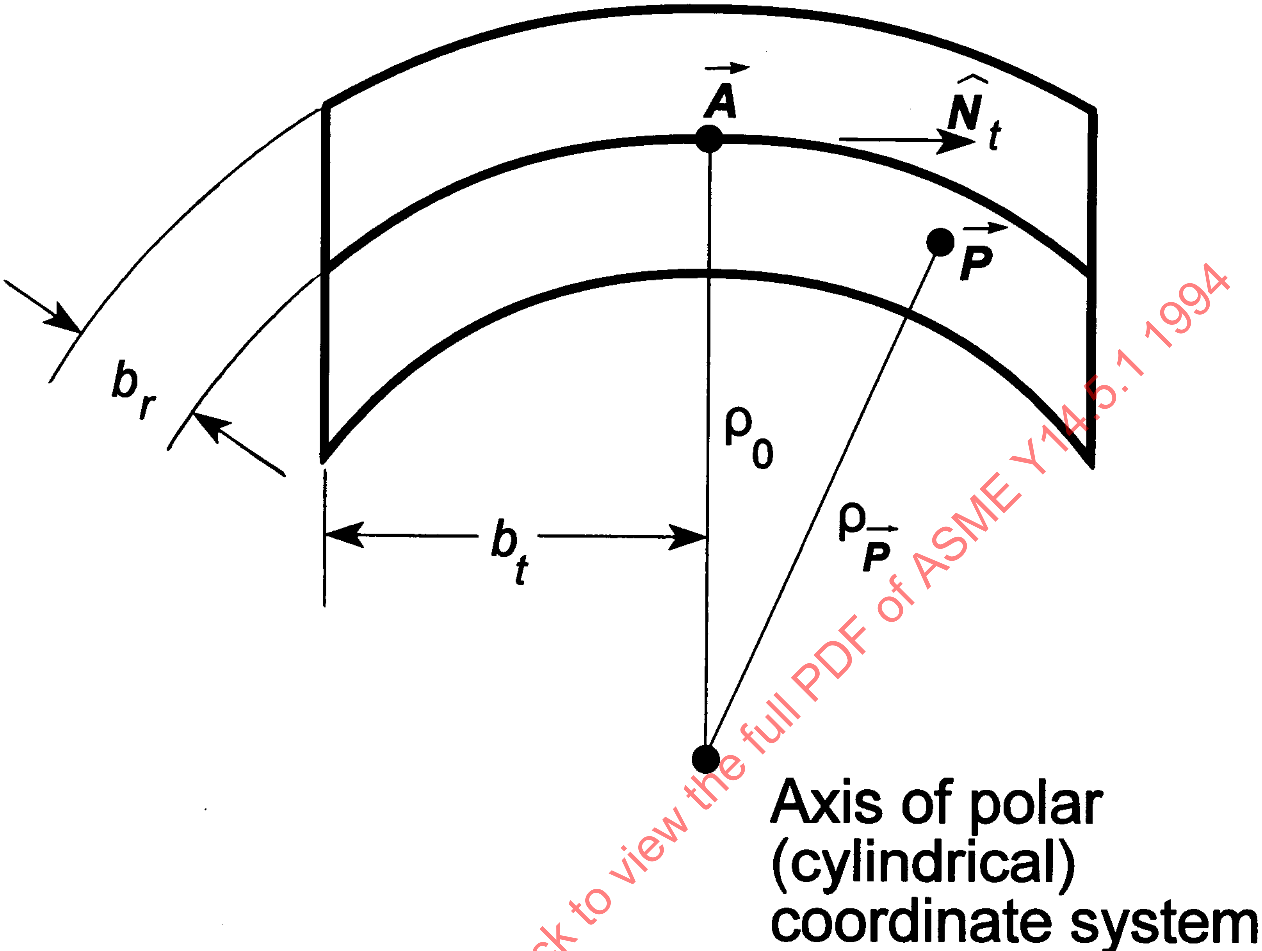


FIG. 5-11 DEFINITION OF THE TOLERANCE ZONE FOR POLAR BIDIRECTIONAL TOLERANCING

b_r = the tolerance zone size parameter for the cylindrical boundaries of the tolerance zone, equal in value to half the difference in radii of the boundaries

b_t = the tolerance zone size parameter for the planar boundaries of the zone, equal in value to half of the distance between the boundaries

The relationship between these quantities is illustrated in Fig. 5-11.

(b) Conformance. A cylindrical feature conforms to a polar, bidirectional position tolerance with radial component t_r and tangential component t_t, each applied at a specified material condition basis, if all points on the axis of the applicable envelope (as determined by the material condition basis) lie within some position zone as defined above with b_r and b_t determined by the appropriate formula from Table 5-11, with $t = t_r$ and $t = t_t$, respectively. Furthermore, the surface must conform to the applicable size limits.

TABLE 5-11 SIZE OF POLAR BIDIRECTIONAL POSITION TOLERANCE ZONE, AXIS INTERPRETATION

		Material Condition Basis		
	b_r or b_t	MMC	RFS	LMC
Feature Type	Internal	$\frac{t}{2} + (r_{AM} - r_{MMC})$	$\frac{t}{2}$	$\frac{t}{2} + (r_{LMC} - r_{AMM})$
	External	$\frac{t}{2} + (r_{MMC} - r_{AM})$	$\frac{t}{2}$	$\frac{t}{2} + (r_{AMM} - r_{LMC})$

(c) Actual value. As with rectangular, bidirectional positional tolerancing, two actual values of position deviation are defined. The actual value of position deviation in either the radial or tangential direction is the thickness of the smallest tolerance zone to which the applicable axis conforms.

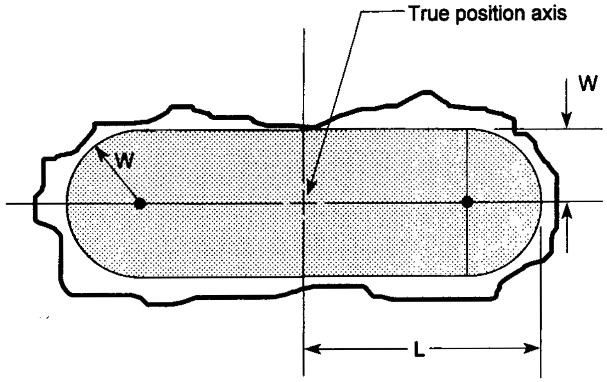


FIG. 5-12 TOLERANCE ZONE AND CONFORMANCE; ELONGATED HOLE AT MMC. THE TOLERANCE ZONE IS A RIGHT CYLINDER SHOWN IN CROSS SECTION

5.6 POSITION TOLERANCING AT MMC FOR BOUNDARIES OF ELONGATED HOLES

An elongated hole is an internal feature consisting of two parallel, opposed, planar faces terminated by cylindrical end caps, tangent to the planar faces, with axes inside the hole. For purposes of positional tolerancing, an elongated hole is considered a feature of size, characterized by two size parameters, its length and width. Positional tolerancing can be applied to elongated holes on an MMC basis. Such tolerancing is always considered to be bidirectional in nature, even if a single tolerance value is applied. Only a surface interpretation is provided.

(a) Definition. For a pattern of elongated holes, a position tolerance at MMC specifies that the surface of each actual hole must not violate the boundary of a corresponding tolerance zone. Each boundary is a right cylinder with an elongated cross section of perfect form as shown in Fig. 5-12. Each boundary is located and oriented by the basic dimensions of the pattern. Each position tolerance zone is the volume interior to the corresponding boundary (the shaded area in Fig. 5-12). The boundary size is characterized by two size parameters, L and W, representing, respectively, the half-length and half-width of the zone.

(b) Conformance. A position tolerance for an elongated hole specifies two values: t_W, controlling position deviation in the direction of the hole width, and t_L, controlling position deviation along the length of the hole. An elongated hole conforms to position tolerances t_W and t_L if all points of the hole surface lie outside a position tolerance zone as defined above, with $W = W_{MMC} - t_W/2$ and $L = L_{MMC} - t_L/2$, where W_{MMC} is the MMC width of the elongated hole and L_{MMC} is the MMC length of the elongated hole.

Furthermore, the hole must surround the tolerance zone and must conform to the limits of size. An elongated hole conforms to the limits of size if there exist two right, elongated-hole cylinders (unconstrained in location or orientation), such that the following conditions hold. One cylinder, with W and L equal to the MMC limits of size, is surrounded by the hole surface. The other cylinder, with W and L equal to the LMC limits of size, surrounds the hole surface.

(c) Actual value. No actual value of position deviation for elongated holes is defined.

5.7 CONCENTRICITY AND SYMMETRY

This Section provides definitions of concentricity and symmetry tolerances that control concentricity and symmetry of features. Concentricity and symmetry controls are similar concepts and are treated together in this Section. Concentricity is that condition where the median points (centroids) of all diametrically opposed elements of a figure of revolution (or correspondingly located elements of two or more radially disposed features) are congruent with a datum axis or center point. Symmetry is that condition where one or more features is equally disposed about a datum plane. A symmetry tolerance is used for the mathematical concept of symmetry about a plane and a concentricity tolerance is used for the mathematical concept of symmetry about a point or symmetry about an axis. Concentricity and symmetry controls are applied to features on an RFS basis only. Datum references must also be RFS.

(a) Definition. A concentricity or symmetry tolerance specifies that the centroid of corresponding point elements on the surfaces of the actual features must lie in some symmetry tolerance zone. The zone is bounded by a sphere, cylinder, or pair of parallel planes of size equal to the total allowable tolerance for the features. The zone is located and oriented by the basic dimensions of the feature(s). The zone is a spherical, cylindrical, or parallel-plane volume defined by all points $\vec{P}$ that satisfy the equation $r(\vec{P}) \leq b$, where b is the radius or half-width of the tolerance zone.

Corresponding point elements are obtained by intersecting a pattern of symmetry rays with the actual feature. The rays of symmetry are determined ac-

TABLE 5-12 SYMMETRY PATTERNS FOR OBTAINING CORRESPONDING FEATURE ELEMENTS

Symmetry Type	Tolerance Type	Patterns of Symmetry Rays
Point	Concentricity	Rays from the datum point
Axis	Concentricity	Rays from, and perpendicular to, the datum axis
Plane	Symmetry	Rays from, and perpendicular to, the datum plane

cording to Table 5-12. If the feature is symmetric about a plane, a two-fold symmetry pattern is always used. For point and axis symmetry, the symmetry pattern is constructed using the lowest order of symmetry of the basic feature. One consequence of this is that surfaces of revolution use two-fold patterns of symmetry rays about the axis or center of symmetry. The feature elements are located at the intersection of the symmetry rays and the actual feature surface.

This principle is illustrated in Fig. 5-13. A feature that has basic three-fold symmetry about a point or (as shown in the figure) an axis results in a three-fold symmetry for the symmetry rays. If the symmetry of the feature is six-fold, however, the symmetry rays are arranged in a two-fold pattern.

(b) Conformance. For a concentricity or symmetry tolerance of t_0, a feature conforms to a symmetry pattern of rays if the centroid of corresponding points of intersection of the rays with the feature all lie within a tolerance zone as defined above with $b = t_0/2$. A feature conforms to a concentricity or symmetry tolerance t_0 if it conforms to symmetry patterns of rays at all possible orientations (for symmetry point), orientations and positions (symmetry axis), or positions (symmetry plane).

(c) Actual value. The actual value of concentricity or symmetry deviation is the smallest tolerance value to which the feature will conform.

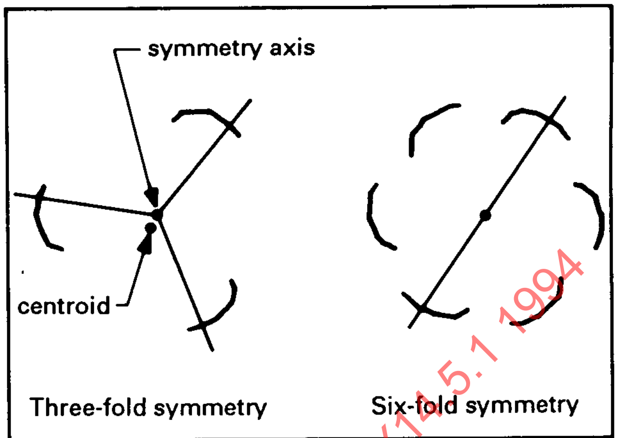


FIG. 5-13 RAYS ARE ARRANGED IN THE LOWEST ORDER OF SYMMETRY ABOUT AN AXIS OR POINT

6 TOLERANCES OF FORM, PROFILE, ORIENTATION, AND RUNOUT

6.1 GENERAL

This Section establishes the principles and methods for mathematical evaluation of ASME Y14.5M-1994 dimensioning and tolerancing which controls form, profile, orientation, and runout of various geometrical shapes and free state variations.

6.2 FORM AND ORIENTATION CONTROL

Form tolerances control straightness, flatness, circularity, and cylindricity. Orientation tolerances control angularity, parallelism, and perpendicularity. A profile tolerance may control form, orientation, size, and location depending on how it is applied.

6.3 SPECIFYING FORM AND ORIENTATION TOLERANCES

Form and orientation tolerances critical to function and interchangeability are specified where the tolerances of size and location do not provide sufficient control. A tolerance of form or orientation may be specified where no tolerance of size is given, for example, the control of flatness after assembly of the parts. A form or orientation tolerance specifies a zone within which the considered feature, its line elements, its axis, or its center plane must be contained. Where the tolerance value represents the diameter of a cylindrical zone, it is preceded by the diameter symbol. In all other cases, the tolerance value represents the total linear distance between two geometric boundaries and no symbol is required.

While the shape of the tolerance zone is well-defined (a cylinder, a zone bounded by two parallel planes, or a zone bounded by two parallel lines), the extent of the tolerance zone (e.g., the length of the cylinder) must also be considered. There are two cases to be considered:

(a) The extent of the tolerance zone is restricted to control a limited area or length of the surface shown by a chain line drawn parallel to the surface profile dimensioned for length and location.

(b) In all other cases, the extent of the tolerance zone is limited to the actual feature surface. For a feature axis, tangent plane, or center plane the extent is defined by projecting the actual surface points onto the axis, tangent plane, or center plane.

6.4 FORM TOLERANCES

Form tolerances are applicable to single (individual) features or elements of single features; therefore, form tolerances are not related to datums. The following subparagraphs cover the particulars of the form tolerances: straightness, flatness, circularity, and cylindricity.

6.4.1 Straightness. *Straightness is a condition where an element of a surface, or an axis, is a straight line. A straightness tolerance specifies a tolerance zone within which the considered element or derived median line must lie. A straightness tolerance is applied in the view where the elements to be controlled are represented by a straight line.*

6.4.1.1 Straightness of a Derived Median Line

(a) Definition. A straightness tolerance for the derived median line of a feature specifies that the derived median line must lie within some cylindrical zone whose diameter is the specified tolerance.

A straightness zone for a derived median line is a cylindrical volume consisting of all points $\vec{P}$ satisfying the condition:

$$|\hat{T} \times (\vec{P} - \vec{A})| \leq \frac{t}{2}$$

where

$\hat{T}$ = the direction vector of the straightness axis

$\vec{A}$ = a position vector locating the straightness axis

t = the diameter of the straightness tolerance zone

(b) Conformance. A feature conforms to a straightness tolerance t_0 if all points of the derived median line lie within some straightness zone as defined above with $t = t_0$. That is, there exist $\hat{T}$ and

$\vec{A}$ such that with $t = t_0$, all points of the derived median line are within the straightness zone.

(c) Actual value. The actual value of straightness for the derived median line of a feature is the smallest straightness tolerance to which the derived median line will conform.

6.4.1.2 Straightness of Surface Line Elements

(a) Definition. A straightness tolerance for the line elements of a feature specifies that each line element must lie in a zone bounded by two parallel lines which are separated by the specified tolerance and which are in the cutting plane defining the line element.

A straightness zone for a surface line element is an area between parallel lines consisting of all points $\vec{P}$ satisfying the condition:

$$|\hat{T} \times (\vec{P} - \vec{A})| \leq \frac{t}{2}$$

and

$$\hat{C}_P \cdot (\vec{P} - \vec{P}_S) = 0$$

$$\hat{C}_P \cdot (\vec{A} - \vec{P}_S) = 0$$

$$\hat{C}_P \cdot \hat{T} = 0$$

where

$\hat{T}$ = the direction vector of the center line of the straightness zone

$\vec{A}$ = a position vector locating the center line of the straightness zone

t = the size of the straightness zone (the separation between the parallel lines)

$\hat{C}_P$ = the normal to the cutting plane defined as being parallel to the cross product of the desired cutting vector and the mating surface normal at $\vec{P}_S$

$\vec{P}_S$ = a point on the surface, contained by the cutting plane

Figure 6-1 illustrates a straightness tolerance zone for surface line elements of a cylindrical feature. Figure 6-2 illustrates a straightness tolerance zone for surface line elements of a planar feature.

(b) Conformance. A surface line element conforms to the straightness tolerance t_0 for a cutting plane if all points of the surface line element lie within some straightness zone as defined above with $t = t_0$. That is, there exist $\hat{T}$ and $\vec{A}$ such that with $t = t_0$, all points of the surface line element are within the straightness zone.

A surface conforms to the straightness tolerance t_0 if it conforms simultaneously for all toleranced surface line elements corresponding to some actual mating surface.

(c) Actual value. The actual value of straightness for a surface is the smallest straightness tolerance to which the surface will conform.

6.4.2 Flatness. *Flatness is the condition of a surface having all elements in one plane. A flatness tolerance specifies a tolerance zone defined by two parallel planes within which the surface must lie.*

(a) Definition. A flatness tolerance specifies that all points of the surface must lie in some zone bounded by two parallel planes which are separated by the specified tolerance.

A flatness zone is a volume consisting of all points $\vec{P}$ satisfying the condition:

$$|\hat{T} \cdot (\vec{P} - \vec{A})| \leq \frac{t}{2}$$

where

$\hat{T}$ = the direction vector of the parallel planes defining the flatness zone

$\vec{A}$ = a position vector locating the mid-plane of the flatness zone

t = the size of the flatness zone (the separation of the parallel planes)

(b) Conformance. A feature conforms to a flatness tolerance t_0 if all points of the feature lie within some flatness zone as defined above, with $t = t_0$. That is, there exist $\hat{T}$ and $\vec{A}$ such that with $t = t_0$, all points of the feature are within the flatness zone.

(c) Actual value. The actual value of flatness for a surface is the smallest flatness tolerance to which the surface will conform.

6.4.3 Circularity (Roundness). *Circularity is a condition of a surface where:*

(a) for a feature other than a sphere, all points of the surface intersected by any plane perpendicular to an axis are equidistant from that axis;

(b) for a sphere, all points of the surface intersected by any plane passing through a common center are equidistant from that center.

A circularity tolerance specifies a tolerance zone bounded by two concentric circles within which each circular element of the surface must lie, and applies independently at any plane described in (a) and (b) above.

(a) Definition. A circularity tolerance specifies that all points of each circular element of the surface

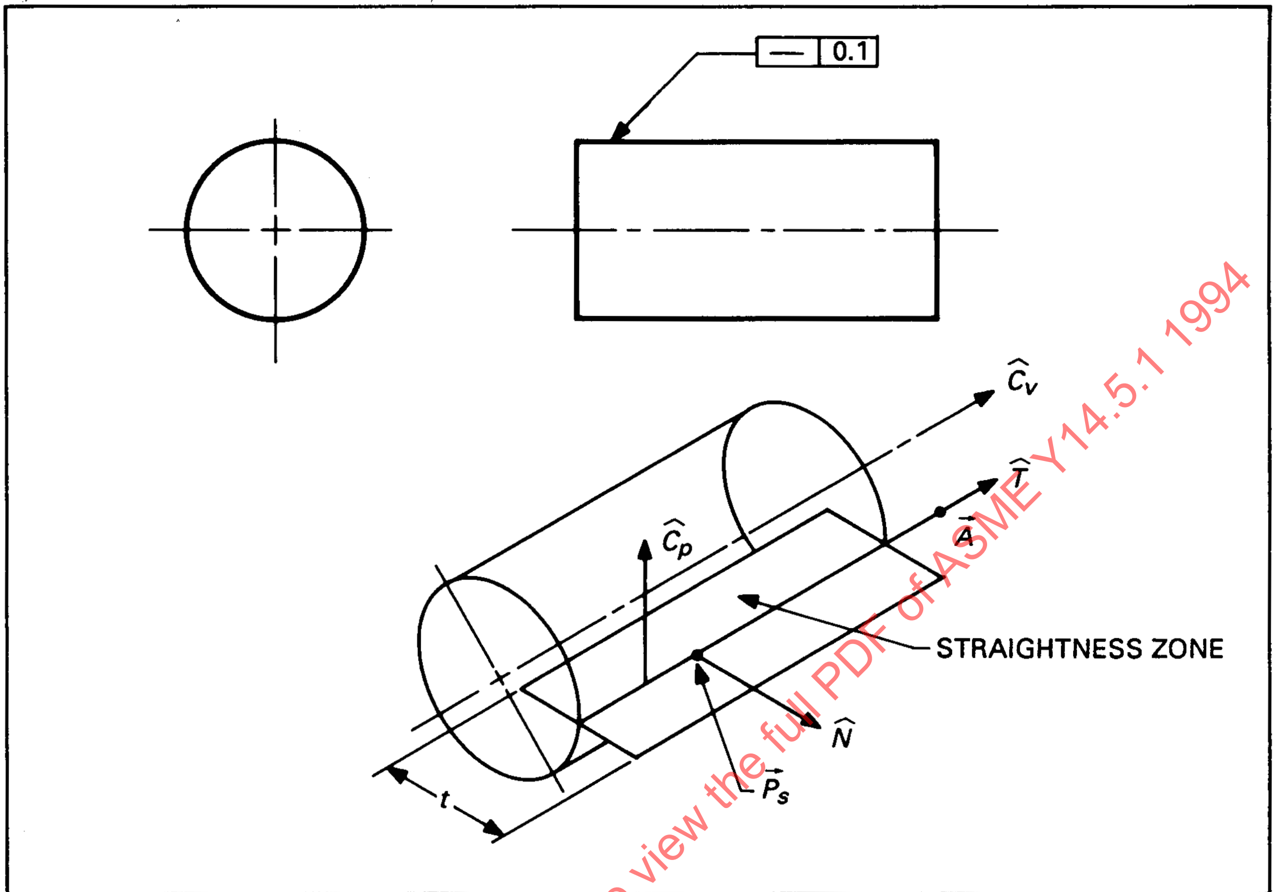


FIG. 6-1 EVALUATION OF STRAIGHTNESS OF A CYLINDRICAL SURFACE

must lie in some zone bounded by two concentric circles whose radii differ by the specified tolerance. Circular elements are obtained by taking cross-sections perpendicular to some spine. For a sphere, the spine is 0-dimensional (a point), and for a cylinder or cone the spine is 1-dimensional (a simple, non self-intersecting, tangent-continuous curve). The concentric circles defining the circularity zone are centered on, and in a plane perpendicular to, the spine.

A circularity zone at a given cross-section is an annular area consisting of all points $\vec{P}$ satisfying the conditions:

$$\hat{T} \cdot (\vec{P} - \vec{A}) = 0$$

and

$$\left| \, |\vec{P} - \vec{A}| - r \, \right| \leq \frac{t}{2}$$

where

$\hat{T}$ = for a cylinder or cone, a unit vector that is tangent to the spine at $\vec{A}$. For a sphere, $\hat{T}$ is a unit vector that points radially in all directions from $\vec{A}$

$\vec{A}$ = a position vector locating a point on the spine

r = a radial distance (which may vary between circular elements) from the spine to the center of the circularity zone ($r > 0$ for all circular elements)

t = the size of the circularity zone

Figure 6-3 illustrates a circularity zone for a circular element of a cylindrical or conical feature.

(b) Conformance. A cylindrical or conical feature conforms to a circularity tolerance t_0 if there exists a 1-dimensional spine such that at each point $\vec{A}$ of the spine the circular element perpendicular to the tangent vector $\hat{T}$ at $\vec{A}$ conforms to the circularity

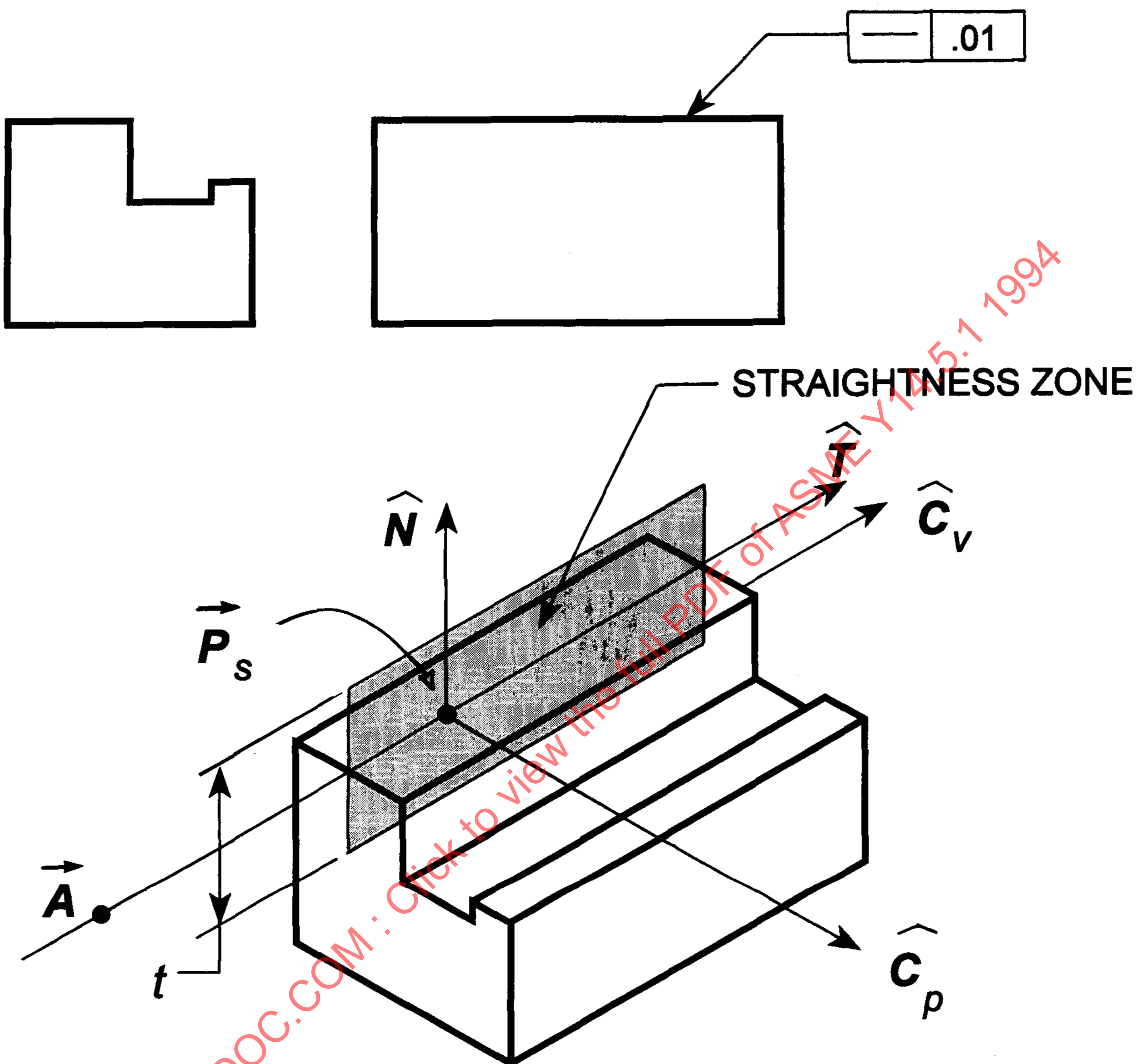


FIG. 6-2 EVALUATION OF STRAIGHTNESS OF A PLANAR SURFACE

tolerance t_0. That is, for each circular element there exist $\vec{A}$ and r such that with $t = t_0$ all points of the circular element are within the circularity zone.

A spherical feature conforms to a circularity tolerance t_0 if there exists a point (a 0-dimensional spine) such that each circular element in each cutting plane containing the point conforms to the circularity tolerance t_0. That is, for each circular element there exist $\hat{T}$, r, and a common $\vec{A}$ such that with $t = t_0$ all points of the circular element are within the circularity zone.

(c) Actual value. The actual value of circularity for a feature is the smallest circularity tolerance to which the feature will conform.

6.4.4 Cylindricity. *Cylindricity is a condition of a surface of revolution in which all points of the surface are equidistant from a common axis. A cylindricity tolerance specifies a tolerance zone bounded by two concentric cylinders within which the surface must lie. In the case of cylindricity, unlike that of circularity, the tolerance applies simultaneously to*

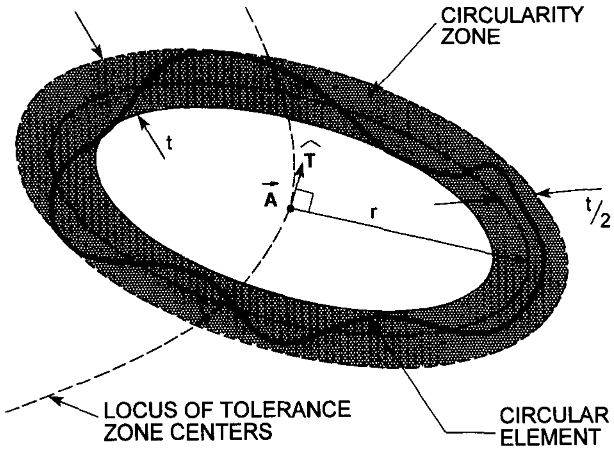


FIG. 6-3 ILLUSTRATION OF CIRCULARITY TOLERANCE ZONE FOR A CYLINDRICAL OR CONICAL FEATURE

both circular and longitudinal elements of the surface (the entire surface).

Note: The cylindricity tolerance is a composite control of form which includes circularity, straightness, and taper of a cylindrical feature.

(a) Definition. A cylindricity tolerance specifies that all points of the surface must lie in some zone bounded by two coaxial cylinders whose radii differ by the specified tolerance.

A cylindricity zone is a volume between two coaxial cylinders consisting of all points $\vec{P}$ satisfying the condition:

$$\left|\,|\hat{T} \times (\vec{P} - \vec{A})| - r\,\right| \le \frac{t}{2}$$

where

$\hat{T}$ = the direction vector of the cylindricity axis
$\vec{A}$ = a position vector locating the cylindricity axis
r = the radial distance from the cylindricity axis to the center of the tolerance zone
t = the size of the cylindricity zone

(b) Conformance. A feature conforms to a cylindricity tolerance t_0 if all points of the feature lie within some cylindricity zone as defined above with $t = t_0$. That is, there exist $\hat{T}$, $\vec{A}$, and r such that with $t = t_0$, all points of the feature are within the cylindricity zone.

(c) Actual value. The actual value of cylindricity for a surface is the smallest cylindricity tolerance to which it will conform.

6.5 PROFILE CONTROL

A profile is the outline of an object in a given plane (two-dimensional figure). Profiles are formed by projecting a three-dimensional figure onto a plane or taking cross sections through the figure. The elements of a profile are straight lines, arcs, and other curved lines. With profile tolerancing, the true profile may be defined by basic radii, basic angular dimensions, basic coordinate dimensions, basic size dimensions, undimensioned drawings, or formulas.

(a) Definition. A profile tolerance zone is an area (profile of a line) or a volume (profile of a surface) generated by offsetting each point on the nominal surface in a direction normal to the nominal surface at that point. For unilateral profile tolerances the surface is offset totally in one direction or the other by an amount equal to the profile tolerance. For bilateral profile tolerances the surface is offset in both directions by a combined amount equal to the profile tolerance. The offsets in each direction may, or may not, be disposed equally.

For a given point $\vec{P}_N$ on the nominal surface there is a unit vector $\hat{N}$ normal to the nominal surface whose positive direction is arbitrary; it may point either into or out of the material. A profile tolerance t consists of the sum of two intermediate tolerances t_+ and t_-. The intermediate tolerances t_+ and t_- represent the amount of tolerance to be disposed in the positive and negative directions of the surface normal $\hat{N}$, respectively, at $\vec{P}_N$. For unilateral profile tolerances either t_+ or t_- equals zero. t_+ and t_- are always non-negative numbers.

The contribution of the nominal surface point $\vec{P}_N$ towards the total tolerance zone is a line segment normal to the nominal surface and bounded by points at distances t_+ and t_- from $\vec{P}_N$. The profile tolerance zone is the union of line segments obtained from each of the points on the nominal surface.

(b) Conformance. A surface conforms to a profile tolerance t_0 if all points $\vec{P}_S$ of the surface conform to either of the intermediate tolerances t_+ or t_- disposed about some corresponding point $\vec{P}_N$ on the nominal surface. A point $\vec{P}_S$ conforms to the intermediate tolerance t_+ if it is between $\vec{P}_N$ and $\vec{P}_N + \hat{N}t_+$. A point $\vec{P}_S$ conforms to the intermediate tolerance t_- if it is between $\vec{P}_N$ and $\vec{P}_N - \hat{N}t_-$. Mathematically, this is the condition that there exists some $\vec{P}_N$ on the nominal surface and some u, $-t_- \le u \le t_+$, for which $\vec{P}_S = \vec{P}_N + \hat{N}u$.

(c) Actual value. For both unilateral and bilateral profile tolerances two actual values are necessarily calculated: one for surface variations in the positive

direction and one for the negative direction. For each direction, the actual value of profile is the smallest intermediate tolerance to which the surface conforms. Note that no single actual value may be calculated for comparison to the tolerance value in the feature control frame, except in the case of unilateral profile tolerances.

6.6 ORIENTATION TOLERANCES

Angularity, parallelism, perpendicularity, and in some instances profile are orientation tolerances applicable to related features. These tolerances control the orientation of features to one another.

In specifying orientation tolerances to control angularity, parallelism, perpendicularity, and in some cases profile, the considered feature is related to one or more datum features. Relation to more than one datum feature is specified to stabilize the tolerance zone in more than one direction.

Tolerance zones are total in value requiring an axis, or all elements of the considered surface to fall within this zone. Where it is a requirement to control only individual line elements of a surface, a qualifying notation, such as EACH ELEMENT or EACH RADIAL ELEMENT, is added to the drawing. This permits control of individual elements of the surface independently in relation to the datum and does not limit the total surface to an encompassing zone.

Where it is desired to control a feature surface established by the contacting points of that surface, the tangent plane symbol is added in the feature control frame after the stated tolerance.

Angularity is the condition of a surface or center plane or axis at a specified angle (other than 90 deg.) from a datum plane or axis.

Parallelism is the condition of a surface or center plane, equidistant at all points from a datum plane or an axis, equidistant along its length from one or more datum planes or a datum axis.

Perpendicularity is the condition of a surface, center plane, or axis at a right angle to a datum plane or axis.

Mathematically, the equations describing angularity, parallelism, and perpendicularity are identical for a given orientation zone type when generalized in terms of the angle(s) between the tolerance zone and the related datum(s). Accordingly, the generic term *orientation* is used in place of angularity, parallelism, and perpendicularity in the definitions. See Appendix A.

An orientation zone is bounded by a pair of parallel planes, a cylindrical surface, or a pair of parallel lines. Each of these cases is defined separately below. If the tolerance value is preceded by the diameter symbol then the tolerance zone is a cylindrical volume; if the notation EACH ELEMENT or EACH RADIAL ELEMENT appears then the tolerance zone is an area between parallel lines; in all other cases the tolerance zone is a volume between parallel planes by default.

6.6.1 Planar Orientation Zone

(a) Definition. An orientation tolerance which is not preceded by the diameter symbol and which does not include the notation EACH ELEMENT or EACH RADIAL ELEMENT specifies that the toleranced surface, center plane, tangent plane, or axis must lie in a zone bounded by two parallel planes separated by the specified tolerance and basically oriented to the primary datum and, if specified, to the secondary datum as well.

A planar orientation zone is a volume consisting of all points $\vec{P}$ satisfying the condition:

$$|\hat{T} \cdot (\vec{P} - \vec{A})| \leq \frac{t}{2}$$

where

$\hat{T}$ = the direction vector of the planar orientation zone
$\vec{A}$ = a position vector locating the mid-plane of the planar orientation zone
t = the size of the planar orientation zone (the separation of the parallel planes)

The planar orientation zone is oriented such that, if $\hat{D}_1$ is the direction vector of the primary datum, then:

$$|\hat{T} \cdot \hat{D}_1| = \begin{cases} |\cos \Theta| & \text{for a primary datum axis} \\ |\sin \Theta| & \text{for a primary datum plane} \end{cases}$$

where Θ is the basic angle between the primary datum and the direction vector of the planar orientation zone.

If a secondary datum is specified, the orientation zone is further restricted to be oriented relative to the direction vector, $\hat{D}_2$, of the secondary datum by:

$$|\hat{T}' \cdot \hat{D}_2| = \begin{cases} |\cos \alpha| & \text{for a secondary datum axis} \\ |\sin \alpha| & \text{for a secondary datum plane} \end{cases}$$

where $\hat{T}'$ is the normalized projection of $\hat{T}$ onto a plane normal to $\hat{D}_1$, and α is the basic angle between the secondary datum and $\hat{T}'$. $\hat{T}'$ is given by:

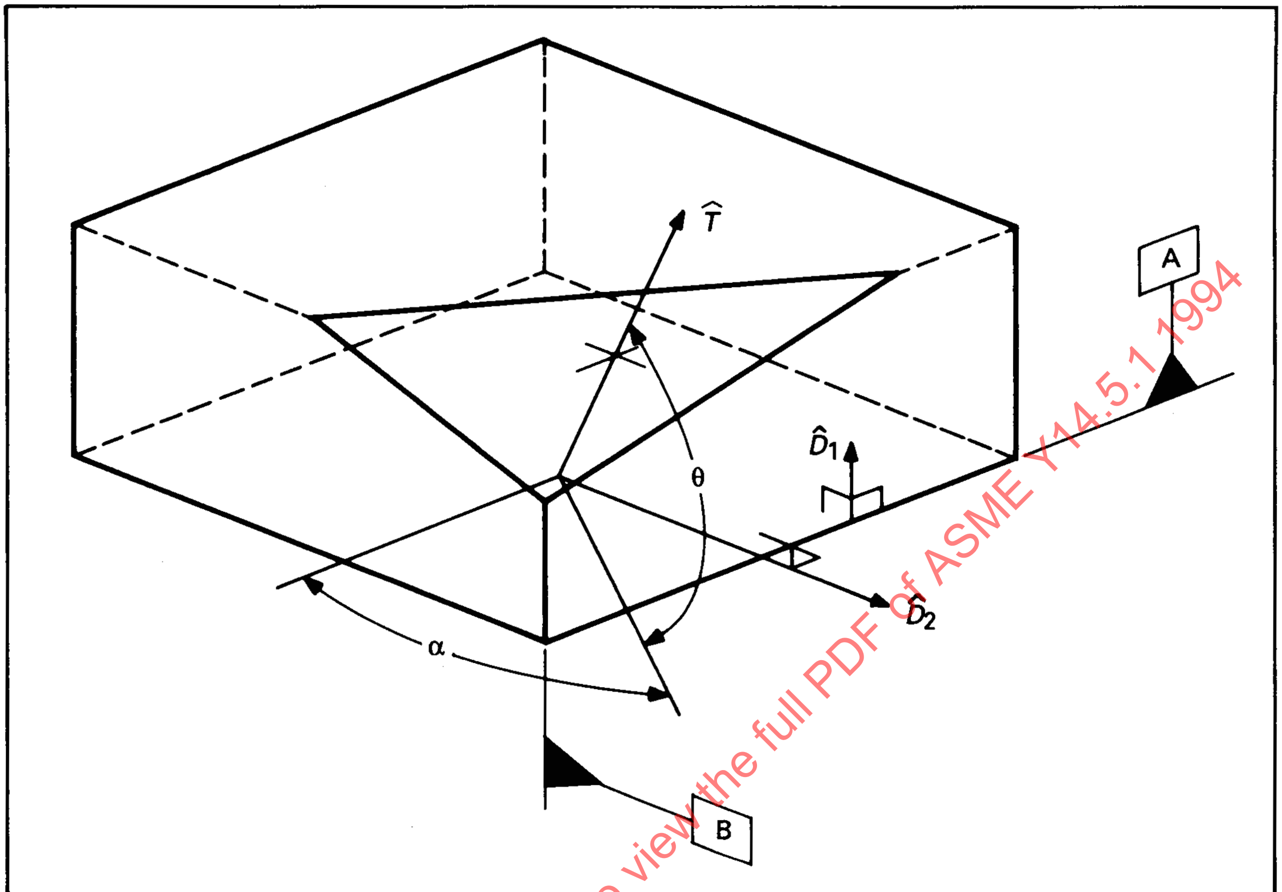


FIG. 6-4 PLANAR ORIENTATION ZONE WITH PRIMARY AND SECONDARY DATUM PLANES SPECIFIED

$$\hat{T}' = \frac{\hat{T} - (\hat{T} \cdot \hat{D}_1)\hat{D}_1}{|\hat{T} - (\hat{T} \cdot \hat{D}_1)\hat{D}_1|}$$

Figure 6-4 shows the relationship of the tolerance zone direction vector to the primary and secondary datums. Figure 6-5 illustrates the projection of $\hat{T}$ onto the primary datum plane to form $\hat{T}'$.

(b) Conformance. A surface, center plane, tangent plane, or axis S conforms to an orientation tolerance t_0 if all points of S lie within some planar orientation zone as defined above with $t = t_0$. That is, there exist $\hat{T}$ and $\vec{A}$ such that with $t = t_0$, all points of S are within the planar orientation zone. Note that if the orientation tolerance refers to both a primary datum and a secondary datum, then $\hat{T}$ is fully determined.

(c) Actual value. The actual value of orientation for S is the smallest orientation tolerance to which S will conform.

6.6.2 Cylindrical Orientation Zone

(a) Definition. An orientation tolerance which is preceded by the diameter symbol specifies that the toleranced axis must lie in a zone bounded by a cylinder with a diameter equal to the specified tolerance and whose axis is basically oriented to the primary datum and, if specified, to the secondary datum as well.

A cylindrical orientation zone is a volume consisting of all points $\vec{P}$ satisfying the condition:

$$|\hat{T} \times (\vec{P} - \vec{A})| \le \frac{t}{2}$$

where

$\hat{T}$ = the direction vector of the axis of the cylindrical orientation zone

$\vec{A}$ = a position vector locating the axis of the cylindrical orientation zone

t = the diameter of the cylindrical orientation zone

The axis of the cylindrical orientation zone is ori-

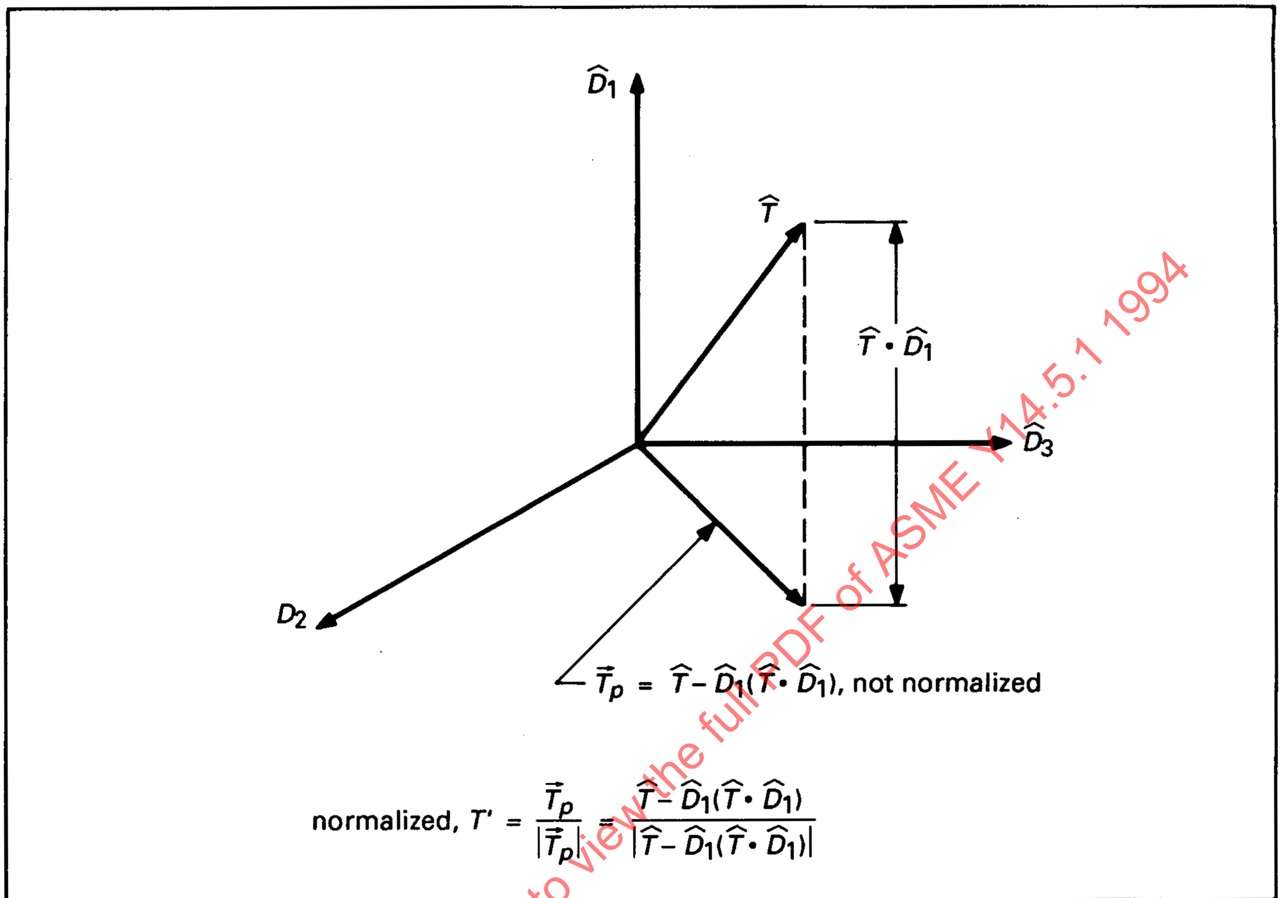


FIG. 6-5 PROJECTION OF TOLERANCE VECTOR ONTO PRIMARY DATUM PLANE

ented such that, if $\hat{D}_1$ is the direction vector of the primary datum, then:

$$|\hat{T} \cdot \hat{D}_1| = \begin{cases} |\cos \Theta| & \text{for a primary datum axis} \\ |\sin \Theta| & \text{for a primary datum plane} \end{cases}$$

where Θ is the basic angle between the primary datum and the direction vector of the axis of the cylindrical orientation zone.

If a secondary datum is specified, the orientation zone is further restricted to be oriented relative to the direction vector, $\hat{D}_2$, of the secondary datum by:

$$|\hat{T}' \cdot \hat{D}_2| = \begin{cases} |\cos \alpha| & \text{for a secondary datum axis} \\ |\sin \alpha| & \text{for a secondary datum plane} \end{cases}$$

where $\hat{T}'$ is the normalized projection of $\hat{T}$ onto a plane normal to $\hat{D}_1$, and α is the basic angle between the secondary datum and $\hat{T}'$. $\hat{T}'$ is given by:

$$\hat{T}' = \frac{\hat{T} - (\hat{T} \cdot \hat{D}_1)\hat{D}_1}{|\hat{T} - (\hat{T} \cdot \hat{D}_1)\hat{D}_1|}$$

Figure 6-6 illustrates a cylindrical orientation tolerance zone.

(b) Conformance. An axis S conforms to an orientation tolerance t_0 if all points of S lie within some cylindrical orientation zone as defined above with $t = t_0$. That is, there exists $\hat{T}$ and $\vec{A}$ such that with $t = t_0$, all points of S are within the orientation zone. Note that if the orientation tolerance refers to both a primary datum and a secondary datum, then $\hat{T}$ is fully determined.

(c) Actual value. The actual value of orientation for S is the smallest orientation tolerance to which S will conform.

6.6.3 Linear Orientation Zone

(a) Definition. An orientation tolerance which includes the notation EACH ELEMENT or EACH RA-

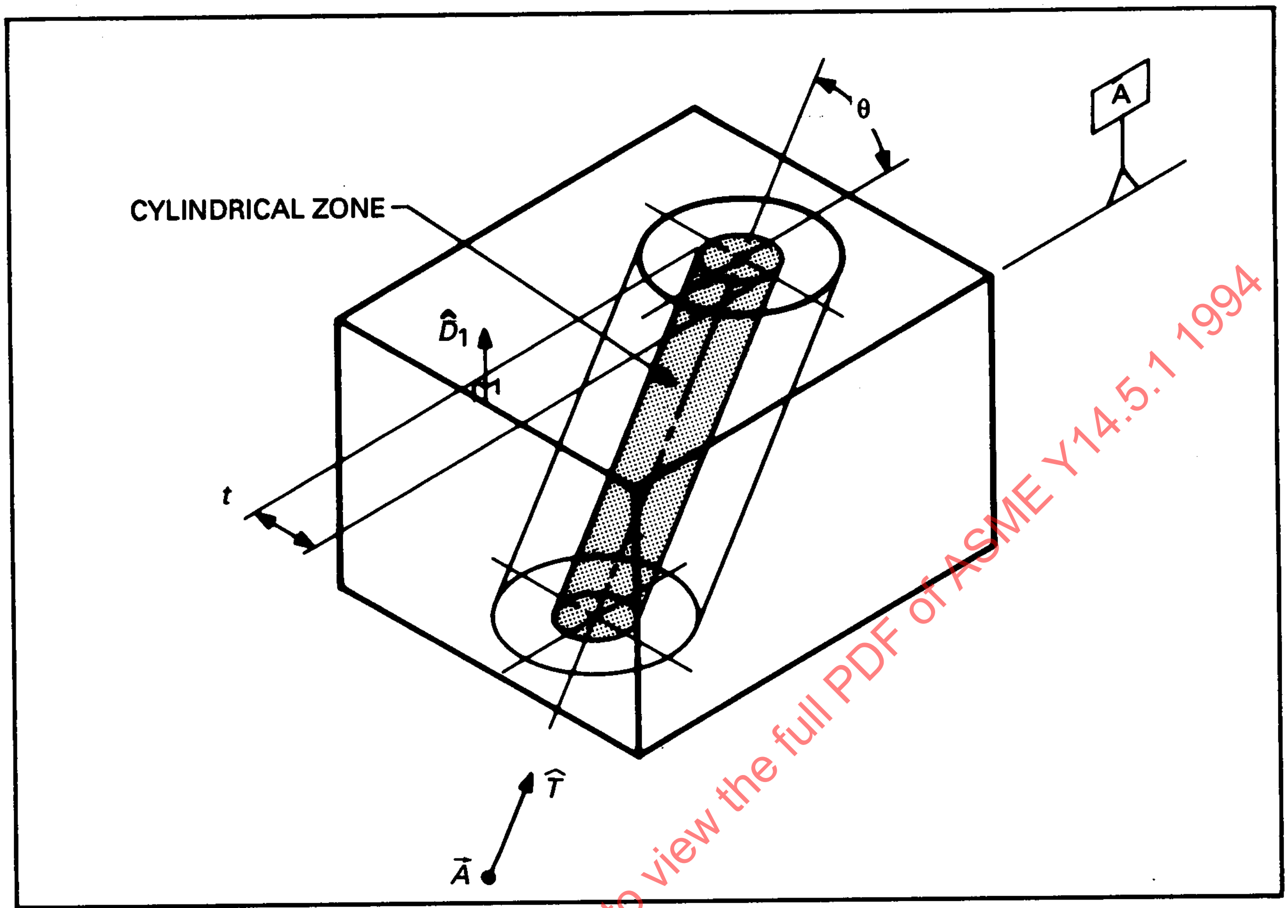


FIG. 6-6 ORIENTATION ZONE BOUNDED BY A CYLINDER WITH RESPECT TO A PRIMARY DATUM PLANE

DIAL ELEMENT specifies that each line element of the toleranced surface must lie in a zone bounded by two parallel lines which are (1) in the cutting plane defining the line element, (2) separated by the specified tolerance, and (3) are basically oriented to the primary datum and, if specified, to the secondary datum as well.

For a surface point $\vec{P}_S$, a linear orientation zone is an area consisting of all points $\vec{P}$ in a cutting plane of direction vector $\hat{C}_P$ that contains $\vec{P}_S$. The points $\vec{P}$ satisfy the conditions:

$$\hat{C}_P \cdot (\vec{P} - \vec{P}_S) = 0$$

and

$$|\hat{T} \times (\vec{P} - \vec{A})| \leq \frac{t}{2}$$

where

$\hat{T}$ = the direction vector of the center line of the linear orientation zone

$\vec{A}$ = a position vector locating the center line of the linear orientation zone

$\vec{P}_S$ = a point on S

$\hat{C}_P$ = the normal to the cutting plane and basically oriented to the datum reference frame

t = the size of the linear orientation zone (the separation between the parallel lines)

The cutting plane is oriented to the primary datum by the constraint:

$$\hat{C}_P \cdot \hat{D}_1 = 0$$

If a secondary datum is specified, the cutting plane is further restricted to be oriented to the direction vector of the secondary datum, $\hat{D}_2$, by the constraint:

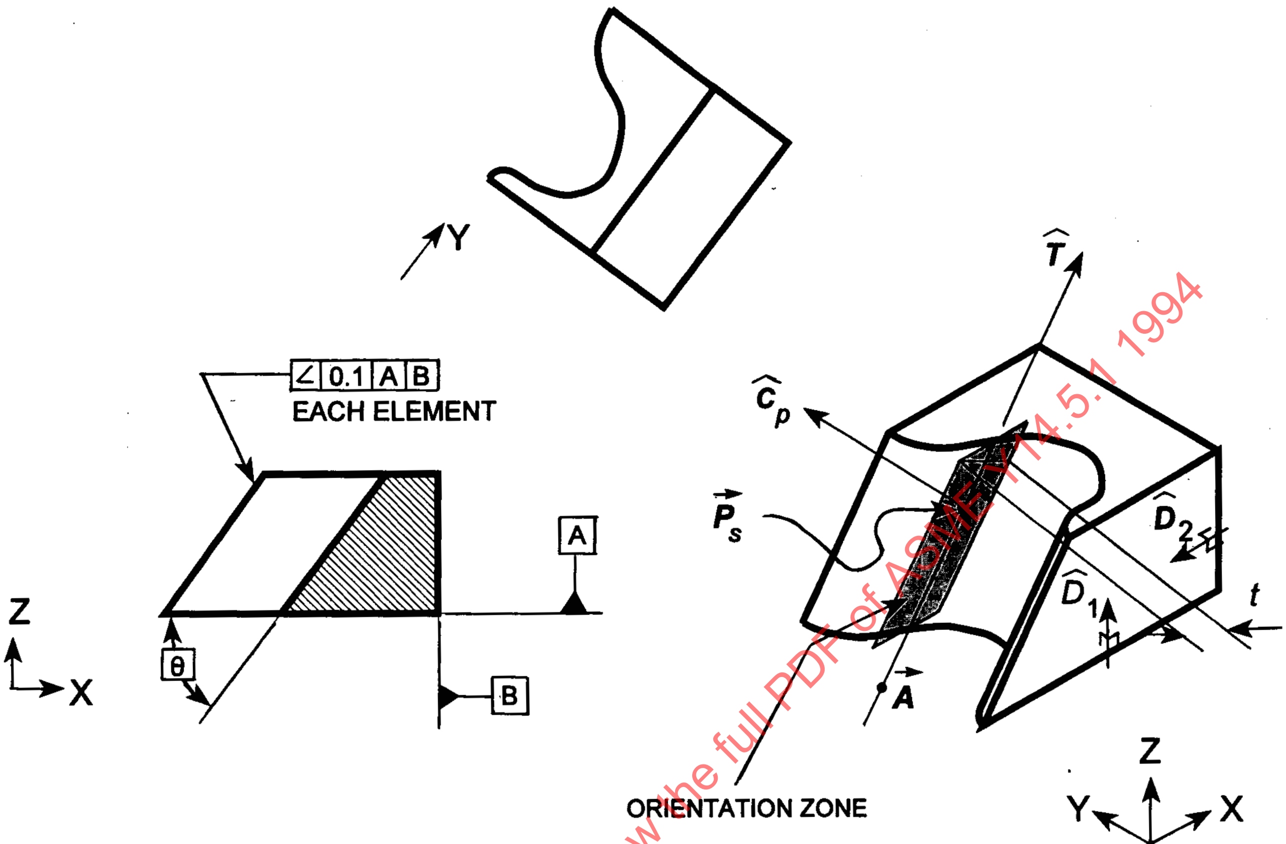


FIG. 6-7 ORIENTATION ZONE BOUNDED BY PARALLEL LINES

$|\hat{C}_P \cdot \hat{D}_2| = |\cos \alpha|$ for a secondary datum axis

$|\hat{C}_P \cdot \hat{D}_2| = |\sin \alpha|$ for a secondary datum plane

The position vector $\vec{A}$, which locates the center line of the linear orientation zone, also locates the cutting plane through the following constraint:

$$\hat{C}_p \cdot (\vec{P}_s - \vec{A}) = 0$$

If a primary or secondary datum axis is specified, and the toleranced feature in its nominal condition is rotationally symmetric about that datum axis, then the cutting planes are further restricted to contain the datum axis as follows:

$$\hat{C}_p \cdot (\vec{P}_s - \vec{B}) = 0$$

where $\vec{B}$ is a position vector that locates the datum axis. Otherwise, the cutting planes are required to be parallel to one another.

The direction vector of the center line of the linear orientation zone, $\hat{T}$, is constrained to lie in the cutting plane by:

$$\hat{C}_P \cdot \hat{T} = 0$$

The center line of the linear orientation zone is oriented such that, if $\hat{D}_1$ is the direction vector of the primary datum, then:

$$|\hat{T} \cdot \hat{D}_1| = \begin{cases} |\cos \Theta| & \text{for a primary datum axis} \\ |\sin \Theta| & \text{for a primary datum plane} \end{cases}$$

where Θ is the basic angle between the primary datum and the direction vector of the linear orientation zone.

Figure 6-7 illustrates an orientation zone bounded by parallel lines on a cutting plane for a contoured surface.

(b) Conformance. A surface, center plane, or tangent plane S conforms to an orientation tolerance t_0 for a cutting plane $\hat{C}_P$ if all points of the intersection of S with $\hat{C}_P$ lie within some linear orientation zone as defined above with $t = t_0$. That is, there exist $\hat{T}$ and $\vec{A}$ such that with $t = t_0$, all points of S are within the linear orientation zone.

A surface S conforms to the orientation tolerance t_0 if it conforms simultaneously for all surface points and cutting planes $\hat{C}_P$.

Note that if the orientation tolerance refers to both a primary datum and a secondary datum, then $\hat{T}$ is fully determined.

(c) Actual value. The actual value of orientation for S is the smallest orientation tolerance to which S will conform.

6.7 RUNOUT TOLERANCE

Runout is a composite tolerance used to control the functional relationship of one or more features of a part to a datum axis. The types of features controlled by runout tolerances include those surfaces constructed around a datum axis and those constructed at right angles to a datum axis.

Surfaces constructed around a datum axis are those surfaces that are either parallel to the datum axis or are at some angle other than 90 deg. to the datum axis. The mathematical definition of runout is necessarily separated into two definitions: one for surfaces constructed around the datum axis, and one for surfaces constructed at right angles to the datum axis. A feature may consist of surfaces constructed both around and at right angles to the datum axis. Separate mathematical definitions describe the controls imposed by a single runout tolerance on the distinct surfaces that comprise such a feature. Circular and total runout are handled in paras. 6.7.1 and 6.7.2 respectively.

Evaluation of runout (especially total runout) on tapered or contoured surfaces requires establishment of actual mating normals. Nominal diameters, and (as applicable) lengths, radii, and angles establish a cross-sectional *desired contour* having perfect form and orientation. The desired contour may be translated axially and/or radially, but may not be tilted or scaled with respect to the datum axis. When a tolerance band is equally disposed about this contour and then revolved around the datum axis, a volumetric tolerance zone is generated.

6.7.1 Circular Runout

6.7.1.1 Surfaces Constructed at Right Angles to a Datum Axis

(a) Definition. The tolerance zone for each circular element on a surface constructed at right angles to a datum axis is generated by revolving a line segment about the datum axis. The line segment is parallel to the datum axis and is of length t_0, where t_0 is the specified tolerance. The resulting tolerance zone is the surface of a cylinder of height t_0.

For a surface point $\vec{P}_S$, a circular runout tolerance zone is the surface of a cylinder consisting of the set of points $\vec{P}$ satisfying the conditions:

$$|\hat{D}_1 \times (\vec{P} - \vec{A})| = r$$

and

$$|\hat{D}_1 \cdot (\vec{P} - \vec{B})| \leq \frac{t}{2}$$

where

r = the radial distance from $\vec{P}_S$ to the axis
$\hat{D}_1$ = the direction vector of the datum axis
$\vec{A}$ = a position vector locating the datum axis
$\vec{B}$ = a position vector locating the center of the tolerance zone
t = the size of the tolerance zone (the height of the cylindrical surface)

(b) Conformance. The circular element through a surface point $\vec{P}_S$ conforms to the circular runout tolerance t_0 if all points of the element lie within some circular runout tolerance zone as defined above with $t = t_0$. That is, there exists $\vec{B}$ such that with $t = t_0$ all points of the surface element are within the circular runout zone.

A surface conforms to the circular runout tolerance if all circular surface elements conform.

(c) Actual value. The actual value of circular runout for a surface constructed at right angles to a datum axis is the smallest circular runout tolerance to which it will conform.

6.7.1.2 Surfaces Constructed Around a Datum Axis

(a) Definition. The tolerance zone for each circular element on a surface constructed around a datum axis is generated by revolving a line segment about the datum axis. The line segment is normal to the desired surface and is of length t_0, where t_0 is the specified tolerance. Depending on the orientation of

the resulting tolerance zone will be either a flat annular area, or the surface of a truncated cone.

For a surface point $\vec{P}_S$, a datum axis $[\vec{A}, \hat{D}_1]$, and a given mating surface, a circular runout tolerance zone for a surface constructed around a datum axis consists of the set of points $\vec{P}$ satisfying the conditions:

$$\frac{\hat{D}_1 \cdot (\vec{P} - \vec{B})}{|\vec{P} - \vec{B}|} = \hat{D}_1 \cdot \hat{N}$$

and

$$||\vec{P} - \vec{B}| - d| \le \frac{t}{2}$$

$$\hat{N} \cdot (\vec{P}_S - \vec{B}) > 0$$

where

$\hat{D}_1$ = the direction vector of the datum axis
$\vec{A}$ = a position vector locating the datum axis
$\hat{N}$ = the surface normal at $\vec{P}_S$ determined from the mating surface
$\vec{B}$ = the point of intersection of the datum axis and the line through $\vec{P}_S$ parallel to the direction vector $\hat{N}$
d = the distance from $\vec{B}$ to the center of the tolerance zone as measured parallel to $\hat{N}$ $(d \ge t/2)$
t = the size of the tolerance zone as measured parallel to $\hat{N}$

Figure 6-8 illustrates a circular runout tolerance zone on a non-cylindrical surface of revolution.

(b) Conformance. The circular element through a surface point $\vec{P}_S$ conforms to the circular runout tolerance t_0 for a given mating surface if all points of the circular element lie within some circular runout tolerance zone as defined above with $t = t_0$. That is, there exists d such that with $t = t_0$ all points of the circular element are within the circular runout tolerance zone.

A surface conforms to a circular runout tolerance t_0 if all circular elements of the surface conform to the circular runout tolerance for the same mating surface.

(c) Actual value. The actual value of circular runout for a surface constructed around a datum axis is the smallest circular runout tolerance to which it will conform.

6.7.2 Total Runout

Surfaces Constructed at Right Angles to a Datum Axis

(a) Definition. A total runout tolerance for a surface constructed at right angles to a datum axis specifies that all points of the surface must lie in a zone bounded by two parallel planes perpendicular to the datum axis and separated by the specified tolerance.

For a surface constructed at right angles to a datum axis, a total runout tolerance zone is a volume consisting of the points $\vec{P}$ satisfying:

$$|\hat{D}_1 \cdot (\vec{P} - \vec{B})| \le \frac{t}{2}$$

where

$\hat{D}_1$ = the direction vector of the datum axis
$\vec{B}$ = a position vector locating the mid-plane of the tolerance zone
t = the size of the tolerance zone (the separation of the parallel planes)

(b) Conformance. A surface conforms to the total runout tolerance t_0 if all points of the surface lie within some total runout tolerance zone as defined above with $t = t_0$. That is, there exists $\vec{B}$ such that with $t = t_0$ all points of the surface are within the total runout zone.

(c) Actual value. The actual value of total runout for a surface constructed at right angles to a datum axis is the smallest total runout tolerance to which it will conform.

Surfaces Constructed Around a Datum Axis

(a) Definition. A total runout tolerance zone for a surface constructed around a datum axis is a volume of revolution generated by revolving an area about the datum axis. This area is generated by moving a line segment of length t_0, where t_0 is the specified tolerance, along the desired contour with the line segment kept normal to, and centered on, the desired contour at each point. The resulting tolerance zone is a volume between two surfaces of revolution separated by the specified tolerance.

Given a datum axis defined by the position vector $\vec{A}$ and the direction vector $\hat{D}_1$, let $\vec{B}$ be a point on the datum axis locating one end of the desired contour, and let r be the distance from the datum axis to the desired contour at point $\vec{B}$. Then, for a given $\vec{B}$ and r, let $C(\vec{B}, r)$ denote the desired contour. (Note: points on this contour can be represented by $[d, r + f(d)]$, where d is the distance along the datum axis from $\vec{B}$.) For each possible $C(\vec{B}, r)$ a total runout

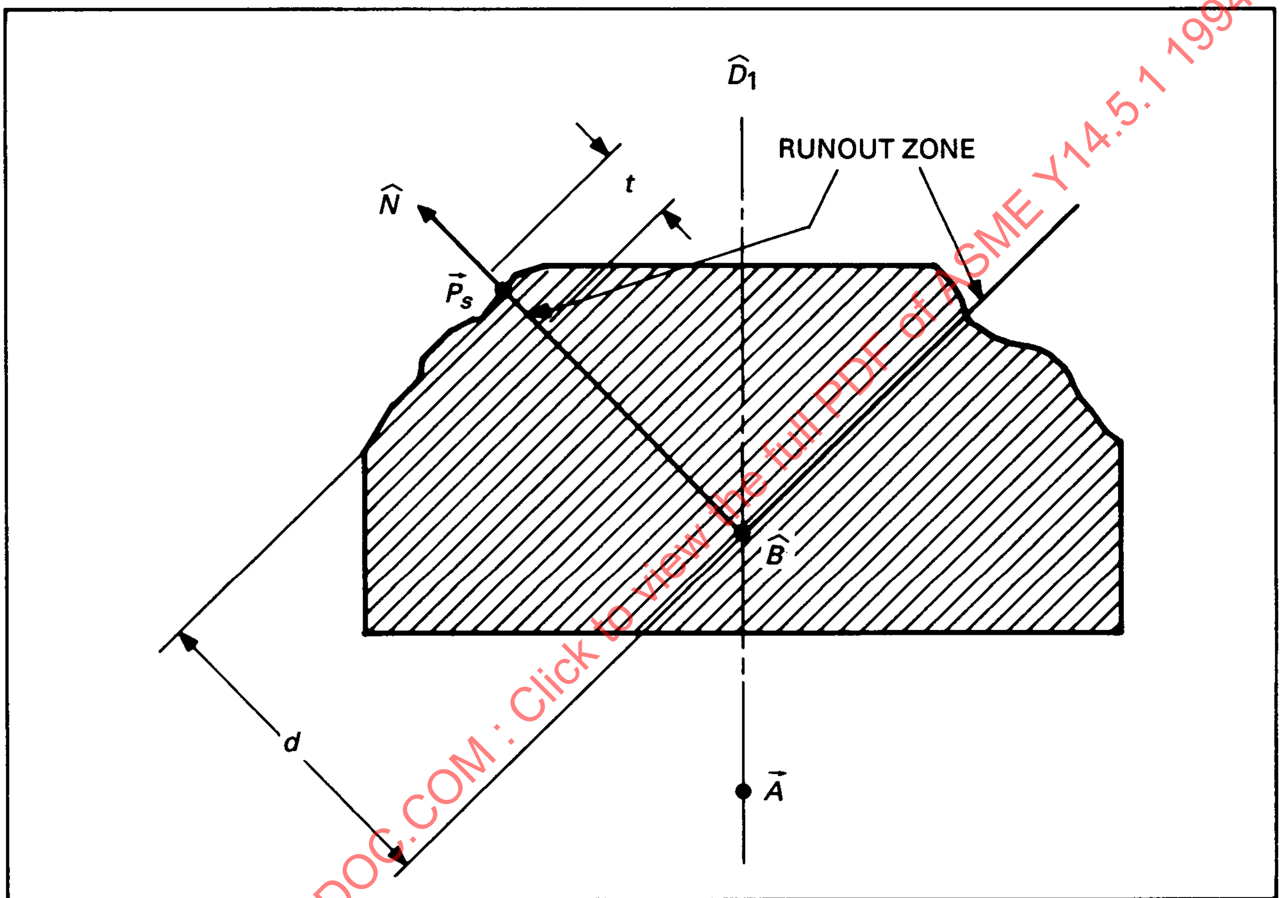


FIG. 6-8 CIRCULAR RUNOUT TOLERANCE ZONE

tolerance zone is defined as the set of points $\vec{P}$ satisfying the condition

$$|\vec{P} - \vec{P}'| \leq \frac{t}{2}$$

where

$\vec{P}'$ = the projection of $\vec{P}$ onto the surface generated by rotating $C(\vec{B}, r)$ about the datum axis

t = the size of the tolerance zone, measured normal to the desired contour

(b) Conformance. A surface conforms to a total runout tolerance t_0 if all points of the surface lie within some total runout tolerance zone as defined above with $t = t_0$. That is, there exist $\vec{B}$ and r such that with $t = t_0$ all points of the surface are within the total runout tolerance zone.

(c) Actual value. The actual value of total runout for a surface constructed around a datum axis is the smallest total runout tolerance to which it will conform.

6.8 FREE STATE VARIATION

Free state variation is a term used to describe distortion of a part after removal of forces applied during manufacture. This distortion is principally due to weight and flexibility of the part and the release of internal stresses resulting from fabrication. A part of this kind, for example, a part with a very thin wall in proportion to its diameter, is referred to as a non-rigid part. In some cases, it may be required that the part meet its tolerance requirements while in the free state. In others, it may be necessary to simulate the mating part interface in order to verify individual or related feature tolerances. This is done by restraining the appropriate features. The restraining forces are those that would be exerted in the assembly or functioning of the part. However, if the dimensions and tolerances are met in the free state, it is usually not necessary to restrain the part unless the effect of subsequent restraining forces on the concerned features could cause other features of the part to exceed specified limits.

APPENDIX A
CONSOLIDATION OF PARALLELISM, PERPENDICULARITY, AND ANGULARITY

(This Appendix is not part of ASME Y14.5.1M-1994.)

A.1 General

Early in the development process of this Standard, one of the more apparent items in Section 6 of ASME Y14.5M-1994 was the fact that the geometric tolerances of parallelism, perpendicularity, and angularity seemed to differ only in the value of the angle orienting the tolerance zone with respect to the datum(s). For parallelism, the angle is 0 deg. for perpendicularity, the angle is 90 deg. and for angularity, the angle may be anything other than 0 deg. and 90 deg. (actually, ASME Y14.5M-1994 excludes only 90 deg. from angularity). In the interest of mathematical brevity and conciseness, the goal was to formally verify this fact, then write a generic mathematical definition to cover all instances of orientation tolerances.

The compilation of a comprehensive list of unique cases of orientation tolerances was the vehicle used to prove the mathematical equivalence of parallelism, perpendicularity, and angularity. This exercise also served to identify applications of orientation tolerances that were not addressed by ASME Y14.5M-1994, and which were later incorporated into ASME Y14.5M-1994 as part of a cooperative effort.

Table A-1 is a chart containing information on 24 unique applications of parallelism, perpendicularity, and angularity. Column one (left-most) segregates according to the three possible orientation tolerance zone types: parallel planes (planar), cylindrical, and parallel lines (linear). Column two identifies the possible feature types to which that tolerance zone type might be applied. Column three separates according to the type of primary datum feature, either a plane or an axis. Column four lists each orientation tolerance. Column five notes the effect of applying a secondary datum to orientation tolerances. (Due to the added effect of a secondary datum for many of the 24 cases, the actual number of unique applications of orientation tolerances is greater than 24.) Column six sequentially lists the cases by number. Column seven contains references to related ASME Y14.5M-1994 paragraphs.

As can be seen from column five of Table A-1, there are four possible situations derived from the application of a secondary datum to an orientation callout. Following is an explanation of each situation.

(a) Allowed. Application of a secondary datum is allowed, and serves to fully restrain the tolerance zone. Where a secondary datum is not applied, the tolerance zone, and hence the toleranced feature, are free to rotate about the direction vector of the primary datum.

(b) Note 1 of Table A-1. The application of a secondary datum imposes no additional control on the toleranced feature.

(c) Note 2 of Table A-1. The omission of a secondary datum will result in what may be an inadequate degree of control of the feature. Refer to the text accompanying the figure for more information.

(d) Note 3 of Table A-1. Without the application of a secondary datum, the control imposed by a particular cylindrical zone callout (case 17) is the same as that for a particular planar zone callout (case 11).

Figures A-1 through A-24 depict minimalist parts that illustrate the types of control exerted by each of the 24 cases of orientation tolerances. Additional figures are presented for the cases that allow the application of secondary datums.

A.2 PLANAR ORIENTATION

Figures A-1 through A-12 depict simple parts that describe the types of control imposed through use of an orientation tolerance with a planar zone relative to a primary datum plane or axis. Where the application of a secondary datum adds additional control, an additional figure is provided to illustrate the effect.

Features that may be controlled by an orientation tolerance with a planar zone are surfaces, center planes, tangent planes, and axes. To avoid lengthy

TABLE A-1 TOLERANCES OF ORIENTATION

Tolerance Zone Boundary	Controlled Element	Datum	Controlled Characteristic	Secondary Control	Case Number	Y14.5M-1994 Reference
Parallel Planes	Feature Surface or Center Plane or Tangent Plane	Axis	Angularity	Allowed	1	6.6.2.1a
			Parallelism	Allowed	2	6.6.3.1a
			Perpendicularity	See note (1)	3	6.6.4.1a
		Plane	Angularity	Allowed	4	6.6.2.1a
			Parallelism	See note (1)	5	6.6.3.1a
			Perpendicularity	Allowed	6	6.6.4.1a
	Feature Axis	Axis	Angularity	See note (2)	7	6.6.2.1b
			Parallelism	See note (2)	8	6.6.3.1b
			Perpendicularity	See note (1)	9	6.6.4.1b
		Plane	Angularity	See note (2)	10	6.6.2.1b
			Parallelism	See note (1)	11	6.6.3.1b
			Perpendicularity	See note (2)	12	No
Cylinder	Feature Axis	Axis	Angularity	Allowed	13	6.6.2.1c
			Parallelism	See note (1)	14	6.6.3.1c
			Perpendicularity	Allowed	15	No
		Plane	Angularity	Allowed	16	6.6.2.1c
			Parallelism	See note (3)	17	6.6.3.1c
			Perpendicularity	See note (1)	18	6.6.4.1c
Parallel Lines	Surface Line Element	Axis	Angularity	See note (2)	19	6.6.2.1d
			Parallelism	Allowed	20	6.6.3.1d
			Perpendicularity	Allowed	21	6.6.4.1d
		Plane	Angularity	See note (2)	22	6.6.2.1d
			Parallelism	Allowed	23	6.6.3.1d
			Perpendicularity	Allowed	24	6.6.4.1d

NOTES:
(1) Secondary datum adds no additional control.
(2) Planar or linear tolerance zone without secondary datum may not provide adequate control.
(3) If no secondary datum is specified, then the result is the same as case 11.

and repetitious text, captions associated with each figure list the toleranced feature as simply a plane or an axis.

A.2.1 Case 1. Figures A-1a and A-1b show two cylindrical parts with one end cut at an angle. In Fig. A-1a the rotational orientation of the angled end about the direction vector of the primary datum axis *A* is of no consequence. Indeed, no other part feature exists to which the toleranced feature (the angled end) can be oriented. Therefore, no secondary datum is specified.

The part depicted in Fig. A-1b has a secondary datum *B* specified, consisting of a hole oriented perpendicular to the primary datum axis. Due to some functional requirement, the rotational orientation of the angled surface with respect to the hole is of some consequence. Therefore, the feature control frame specifies a secondary datum.

Note that the angle α is 0 deg. here. α is the basic angle between the secondary datum and the normalized projection of the direction vector of the planar orientation zone onto the plane perpendicular to the direction vector of the primary datum axis.

A.2.2 Case 2. Figure A-2a shows a cylindrical part with a flat surface toleranced to be parallel to the datum axis between parallel planes separated by a specified tolerance. Since no further restriction on orientation is desired, no secondary datum has been specified.

The part in Fig. A-2b differs from that in Fig. A-2a in that a feature exists to which the toleranced surface can be oriented. The hole perpendicular to the primary datum axis is datum *B* and serves to further orient the tolerance zone.

A.2.3 Case 3. Figure A-3 shows a cylindrical part with an end face toleranced to be perpendicular to the datum axis between parallel planes separated by a specified tolerance. Notice that there is no way to impose additional orientation control on the toler-

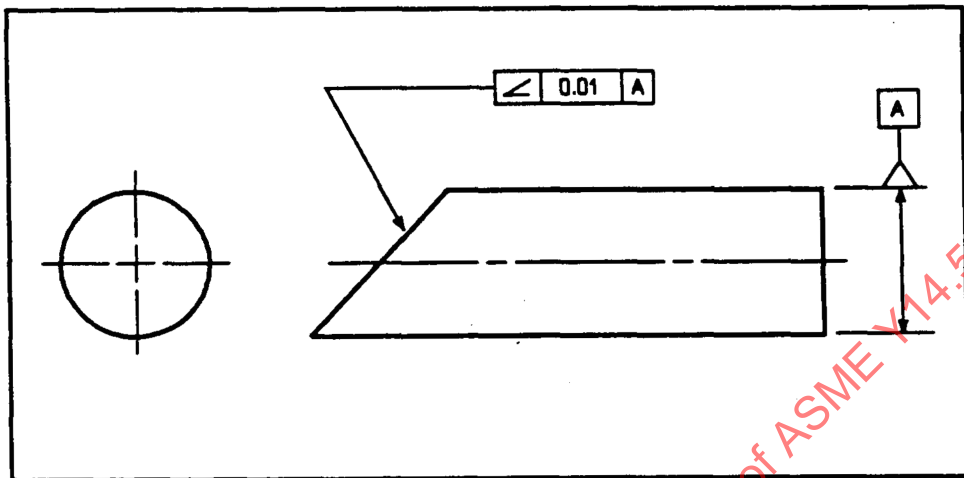


FIG. A-1a PLANAR ANGULARITY OF A PLANE; PRIMARY DATUM AXIS; NO SECONDARY DATUM SPECIFIED

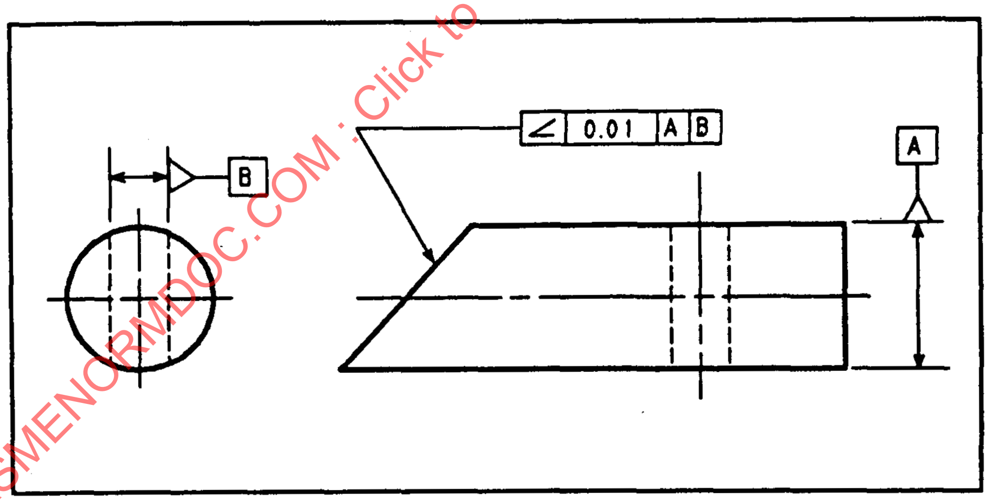


FIG. A-1b PLANAR ANGULARITY OF A PLANE; PRIMARY DATUM AXIS; SECONDARY DATUM AXIS CONTROLS ROTATION OF THE TOLERANCE ZONE ABOUT THE PRIMARY DATUM AXIS

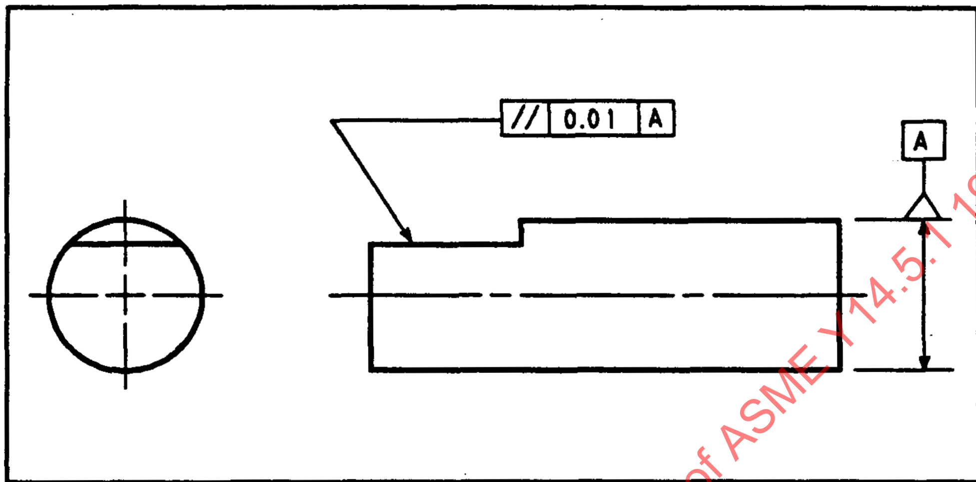


FIG. A-2a PLANAR PARALLELISM OF A PLANE; PRIMARY DATUM AXIS; NO SECONDARY DATUM SPECIFIED

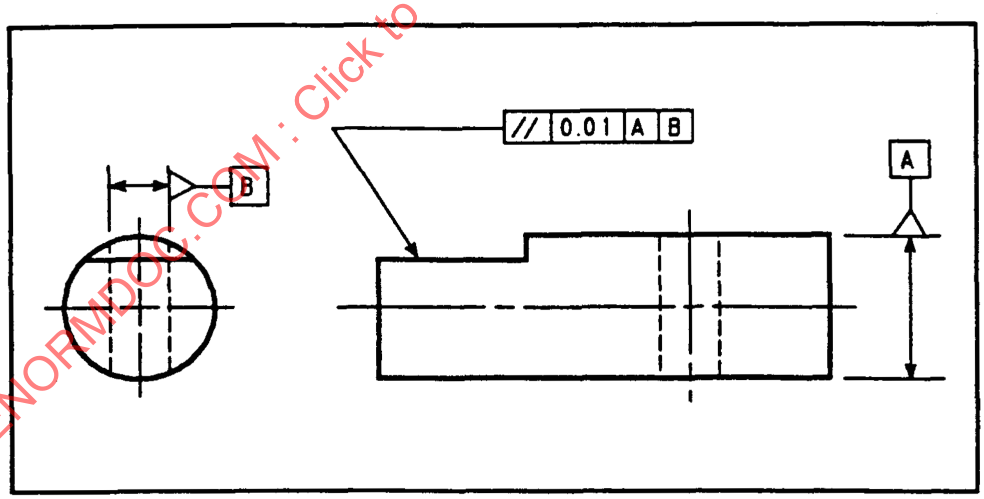


FIG. A-2b PLANAR PARALLELISM OF A PLANE; PRIMARY DATUM AXIS; SECONDARY DATUM PLANE CONTROLS ROTATION OF THE TOLERANCE ZONE ABOUT THE PRIMARY DATUM AXIS

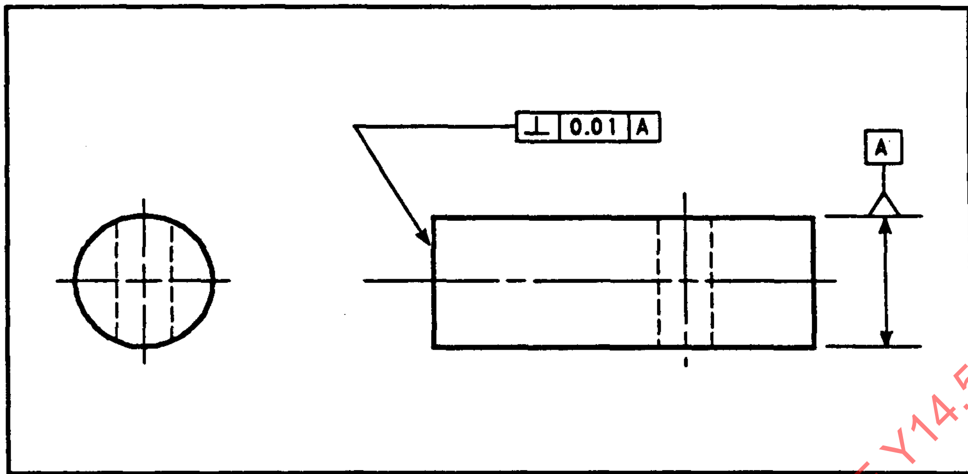


FIG. A-3 PLANAR PERPENDICULARITY OF A PLANE; PRIMARY DATUM AXIS; NO SECONDARY DATUM SPECIFIED. A SECONDARY DATUM WOULD PROVIDE NO ADDITIONAL CONTROL

anced feature. Therefore, no secondary datum is specified.

A.2.4 Case 4. Figure A-4a shows a disk with a beveled notch which is toleranced for angularity with respect to the primary datum plane between parallel planes separated by a specified tolerance. No further restriction on orientation is needed, and neither is it possible for this part. Therefore, no secondary datum has been specified.

Figure A-4b shows a block-type part with an angled face which is toleranced for angularity with respect to the primary datum plane between parallel planes separated by a specified tolerance. Unlike the disk in Fig. A-4a, this part enables a further restriction on orientation of the tolerance zone by virtue of its flat sides.

A.2.5 Case 5. Figure A-5 shows a block whose top surface is toleranced to be parallel to primary datum plane *A* between parallel planes separated by a specified tolerance. Note that, despite the presence of flat sides perpendicular to the primary datum plane, i.e., potential secondary datums, there is no means to further restrict orientation in this case.

A.2.6 Case 6. Figure A-6a shows a disk with an edge sliced off. As indicated, the toleranced surface is to be perpendicular to primary datum plane *A* between parallel planes separated by a specified tolerance. No secondary datum is specified because no further restriction on orientation is desired.

Figure A-6b shows a block with one side face toleranced to be perpendicular to the primary datum plane between parallel planes separated by a specified tolerance. Due to part function considerations, the tolerance zone is further restricted to have a particular orientation with respect to a secondary datum. Thus, a secondary datum has been specified in the feature control frame.

A.2.7 Case 7. Figure A-7a shows a cylindrical part with a hole. The axis of the hole is toleranced for angularity with respect to the primary datum axis between parallel planes separated by a specified tolerance. No other orientation of the hole's axis is critical to this part's function. Therefore, no secondary datum is specified. Note that since the tolerance zone consists of parallel planes between which the axis of the hole must be contained, a wide range of orientations of the hole is permissible. This includes any hole whose angle with the direction vector of the primary datum is greater than the basic angle. Since this lack of control may not be desired, care should be exercised in applying a planar orientation zone to an axis.

Figure A-7b shows a part similar to Fig. A-7a, with the difference being the application of a partial flat as a secondary datum. Unlike Fig. A-7a, this part's function requires a particular orientation of the tolerance zone with respect to the flat. This restriction serves to limit the wide range of acceptability prevalent in Fig. A-7a.

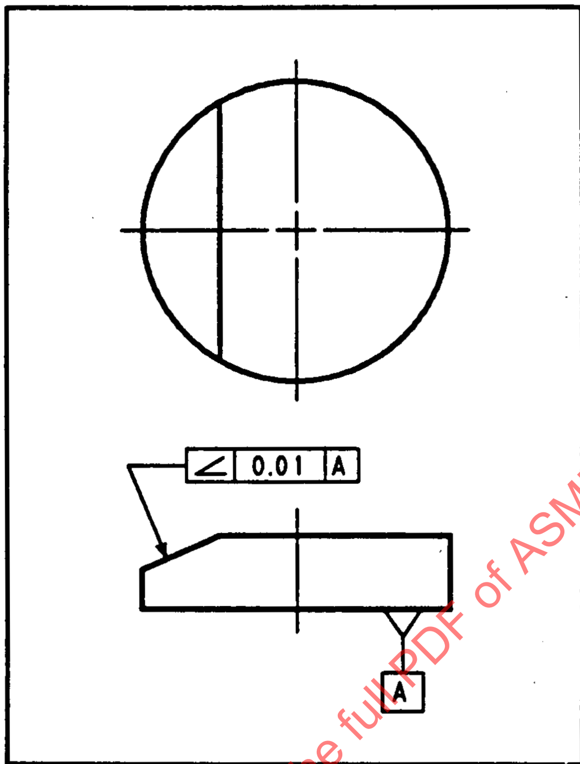


FIG. A-4a PLANAR ANGULARITY OF A PLANE; PRIMARY DATUM PLANE; NO SECONDARY DATUM SPECIFIED

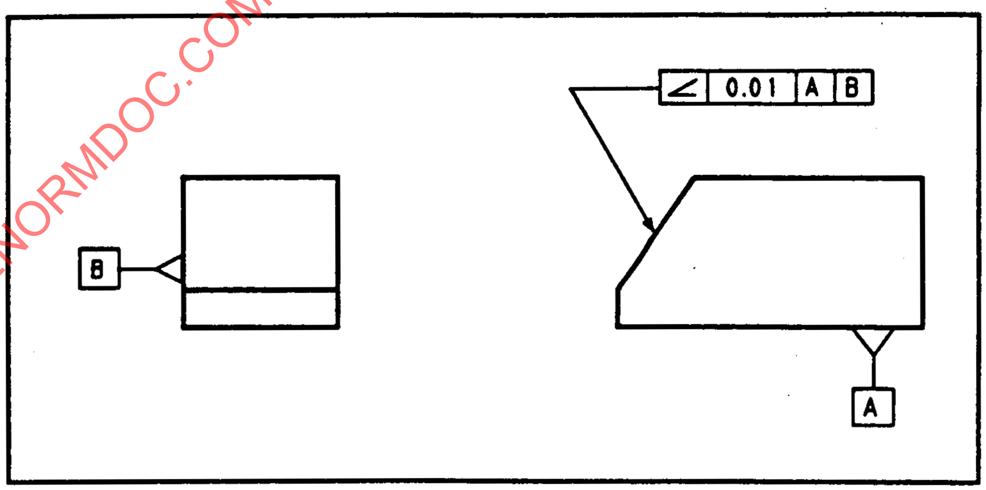


FIG. A-4b PLANAR ANGULARITY OF A PLANE; PRIMARY DATUM PLANE; SECONDARY DATUM PLANE CONTROLS ROTATION OF THE TOLERANCE ZONE ABOUT THE PRIMARY DATUM PLANE

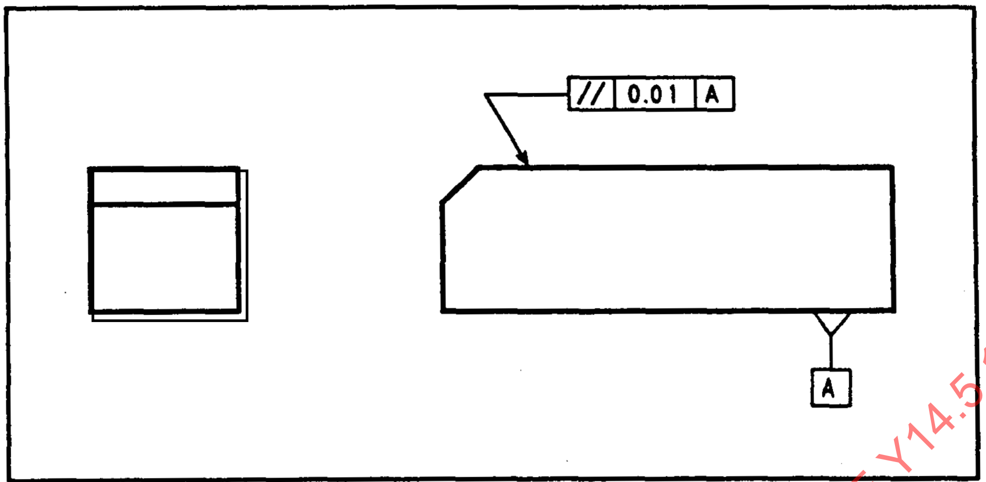


FIG. A-5 PLANAR PARALLELISM OF A PLANE; PRIMARY DATUM PLANE; NO SECONDARY DATUM SPECIFIED. A SECONDARY DATUM WOULD PROVIDE NO ADDITIONAL CONTROL

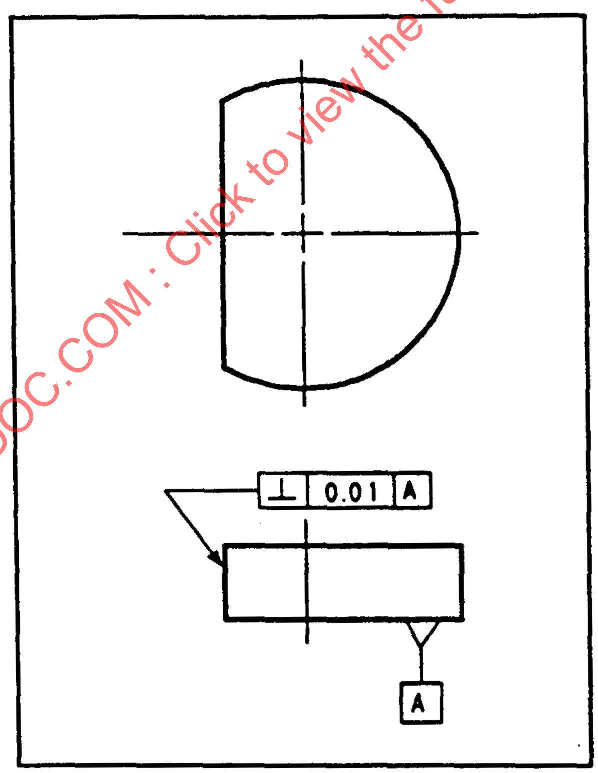


FIG. A-6a PLANAR PERPENDICULARITY OF A PLANE; PRIMARY DATUM PLANE; NO SECONDARY DATUM SPECIFIED

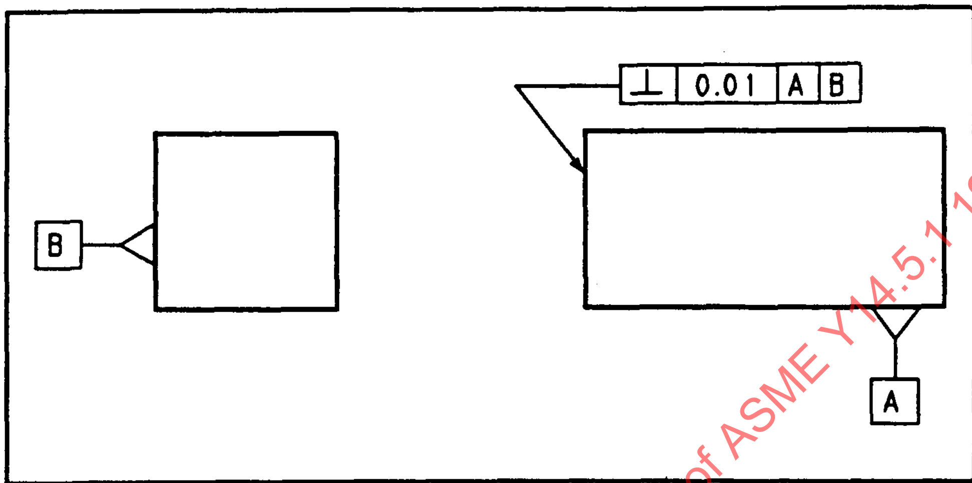


FIG. A-6b PLANAR PERPENDICULARITY OF A PLANE; PRIMARY DATUM PLANE; SECONDARY DATUM PLANE CONTROLS ROTATION OF THE TOLERANCE ZONE ABOUT THE PRIMARY DATUM PLANE

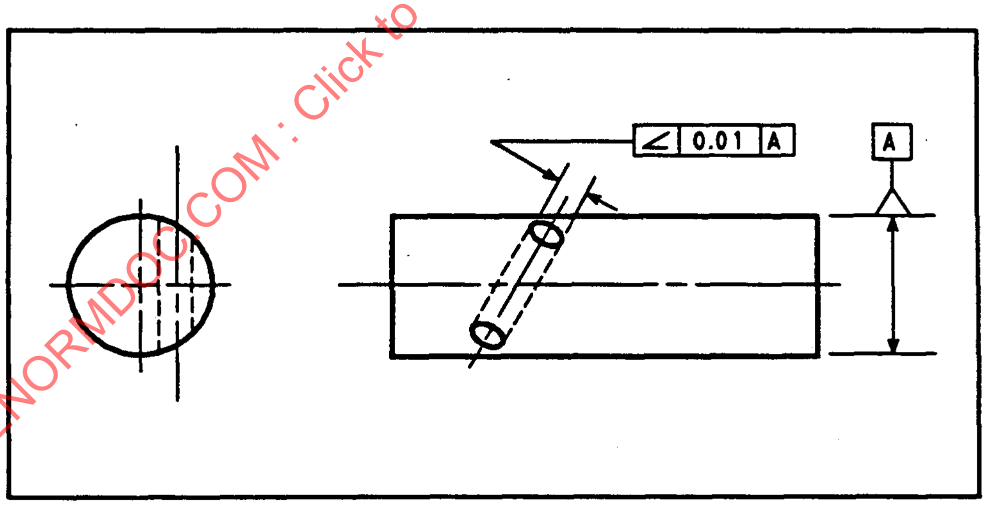


FIG. A-7a PLANAR ANGULARITY OF AN AXIS; PRIMARY DATUM AXIS; NO SECONDARY DATUM SPECIFIED. LACK OF A SECONDARY DATUM MAY NOT PROVIDE ADEQUATE CONTROL

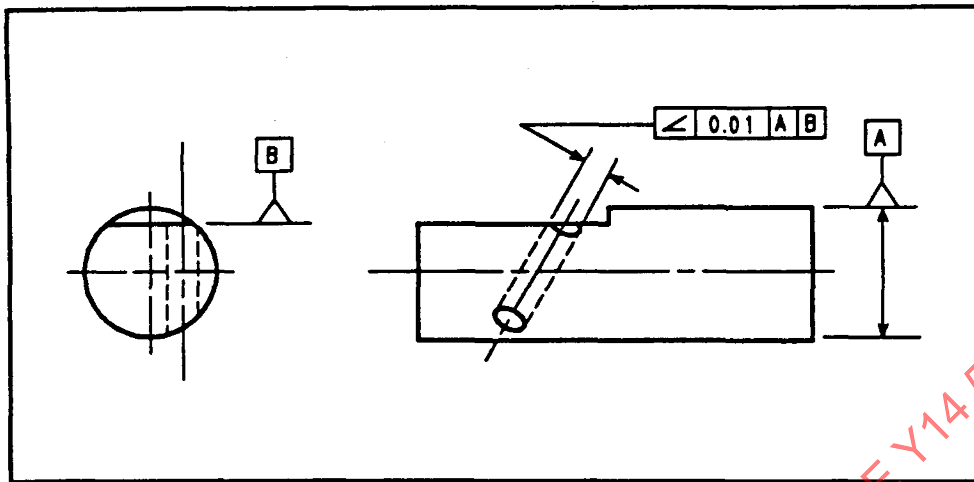


FIG. A-7b PLANAR ANGULARITY OF AN AXIS; PRIMARY DATUM AXIS; SECONDARY DATUM PLANE CONTROLS ROTATION OF THE TOLERANCE ZONE ABOUT THE PRIMARY DATUM AXIS

A.2.8 Case 8. Figure A-8a shows a cylindrical part with a hole whose axis is toleranced to be parallel to the primary datum axis between parallel planes separated by a specified tolerance. Without the application of a secondary datum, this tolerance exerts no control whatsoever on the orientation of the hole's axis.

Figure A-8b shows a part similar to Fig. A-8a, with the difference being the application of a partial flat as a secondary datum. The application of a secondary datum serves to restrict the unlimited range of acceptability prevalent in Fig. A-8a.

A.2.9 Case 9. Figure A-9 shows a cylindrical part with a hole whose axis is toleranced for perpendicularity between parallel planes with respect to the primary datum axis. Rotation of this zone about the datum axis does not change the tolerance zone. Hence, use of a secondary datum would not exert a further restriction.

A.2.10 Case 10. Figure A-10a shows a disk with a hole whose axis is toleranced for angularity between parallel planes with respect to the primary datum plane. As such, the tolerance zone can be rotated such that any hole whose axis is at a smaller angle with respect to the primary datum plane than the basic angle can be brought within this tolerance specification.

Figure A-10b shows a similar tolerance specification as in Fig. A-10a, with the difference being the application of a secondary datum. The application of a secondary datum prevents any rotation of the tolerance zone.

A.2.11 Case 11. Figure A-11 shows a block with a hole whose axis is toleranced for parallelism between parallel planes with respect to a primary datum plane. The application of a secondary datum would not further restrict the orientation of the tolerance zone.

A.2.12 Case 12. Figure A-12a shows a disk toleranced for perpendicularity between parallel planes with respect to a primary datum plane. The tolerance zone can be rotated such that any hole can be brought within this tolerance specification, including holes that are parallel to the primary datum plane.

Figure A-12b shows a similar tolerance specification as in Fig. A-12a, with the difference being the application of a secondary datum. The application of the secondary datum prevents any rotation of the tolerance zone.

A.3 CYLINDRICAL ORIENTATION

Figures A-13 through A-18 depict simple parts that describe the types of control imposed through use of an orientation tolerance with a cylindrical zone relative to a primary datum plane or axis. Where the application of a secondary datum adds additional control, an additional figure is provided to illustrate the effect.

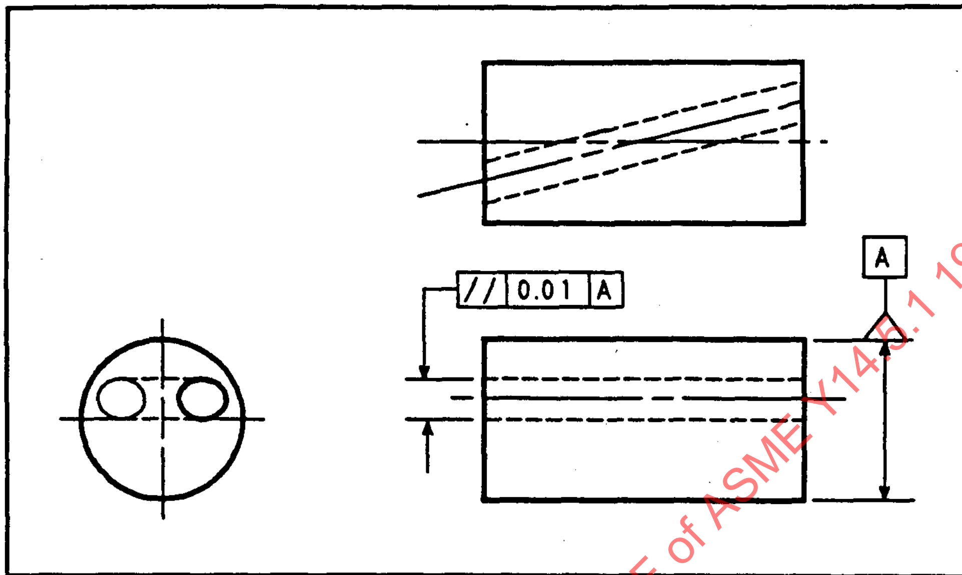


FIG. A-8a PLANAR PARALLELISM OF AN AXIS; PRIMARY DATUM AXIS; NO SECONDARY DATUM SPECIFIED. LACK OF A SECONDARY DATUM MAY NOT PROVIDE ADEQUATE CONTROL

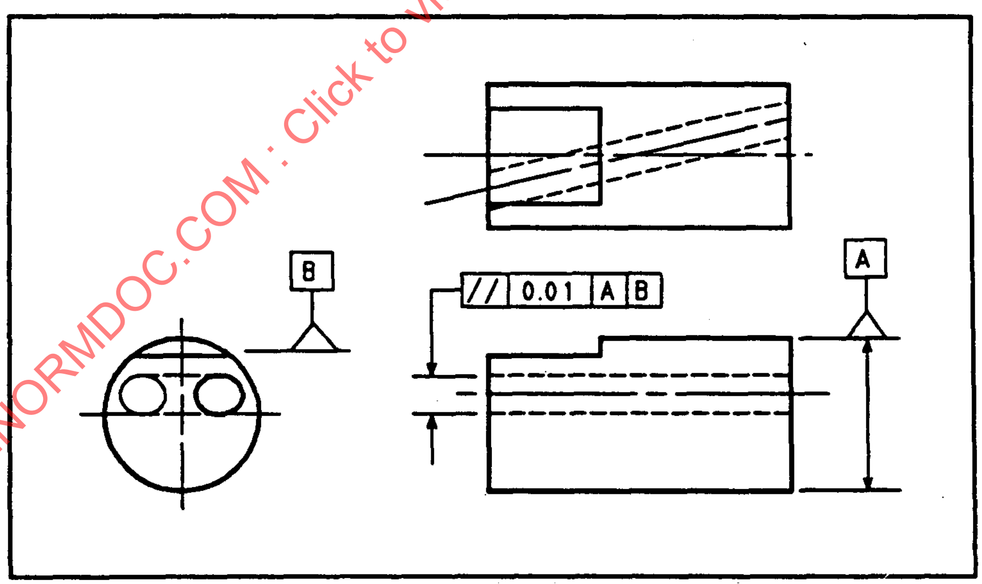


FIG. A-8b PLANAR PARALLELISM OF AN AXIS; PRIMARY DATUM AXIS; SECONDARY DATUM PLANE CONTROLS ROTATION OF THE TOLERANCE ZONE ABOUT THE PRIMARY DATUM AXIS

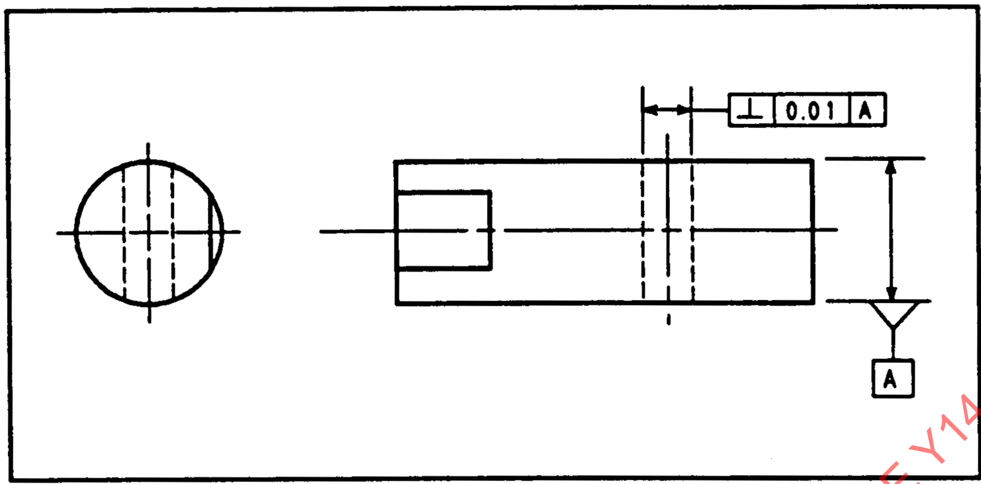


FIG. A-9 PLANAR PERPENDICULARITY OF AN AXIS; PRIMARY DATUM AXIS; NO SECONDARY DATUM SPECIFIED. A SECONDARY DATUM WOULD PROVIDE NO ADDITIONAL CONTROL

Only axes may be controlled by an orientation tolerance with a cylindrical zone.

A.3.1 Case 13. Figure A-13a shows a cylindrical part with a hole whose axis is toleranced for angularity within a cylindrical zone with respect to a primary datum axis. This tolerance zone is free to rotate about the primary datum axis.

Figure A-13b shows a similar part as in Fig. A-13a, with the difference being the application of a secondary datum. Application of the secondary datum prevents rotation of the tolerance zone about the primary datum axis, hence fully restricts the orientation of the tolerance zone.

A.3.2 Case 14. Figure A-14 shows a cylindrical part with a hole toleranced for parallelism within a cylindrical zone with respect to a primary datum axis. Since the orientation of this tolerance zone cannot be further restricted, the application of a secondary datum would provide no additional control.

A.3.3 Case 15. Figure A-15a shows a cylindrical part with a hole toleranced for perpendicularity within a cylindrical zone with respect to a primary datum axis. The tolerance zone is free to rotate about the primary datum axis.

Figure A-15b shows a similar part as in Fig. A-15a, with the difference being the application of a secondary datum. The specification shown requires that the axis of the tolerance zone be parallel to the secondary datum. This prevents rotation of the tolerance zone about the primary datum axis.

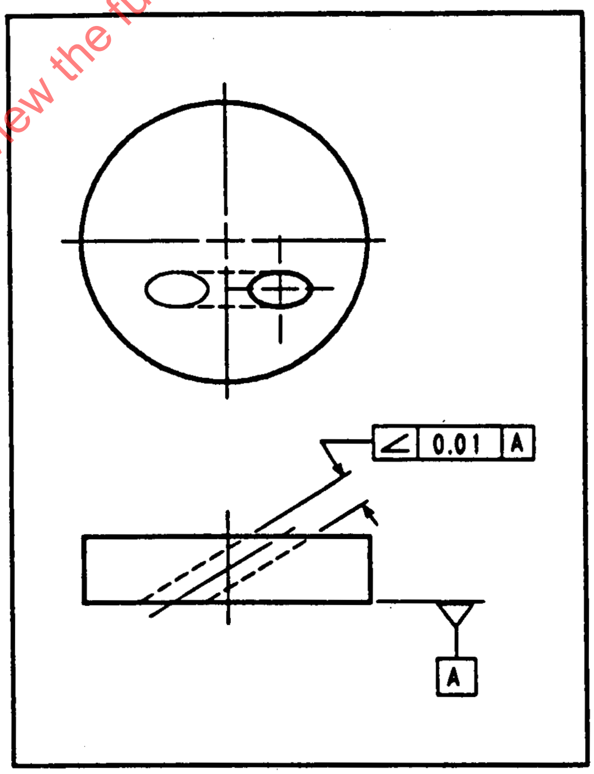


FIG. A-10a PLANAR ANGULARITY OF AN AXIS; PRIMARY DATUM PLANE; NO SECONDARY DATUM SPECIFIED

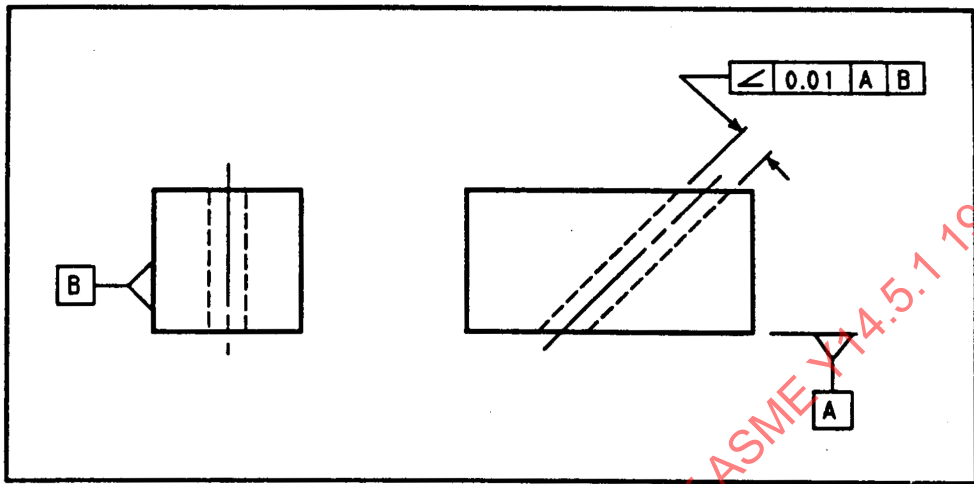


FIG. A-10b PLANAR ANGULARITY OF AN AXIS; PRIMARY AND SECONDARY DATUM PLANES; ZONE ROTATION IS CONTROLLED ABOUT THE DIRECTION VECTOR OF THE PRIMARY DATUM PLANE

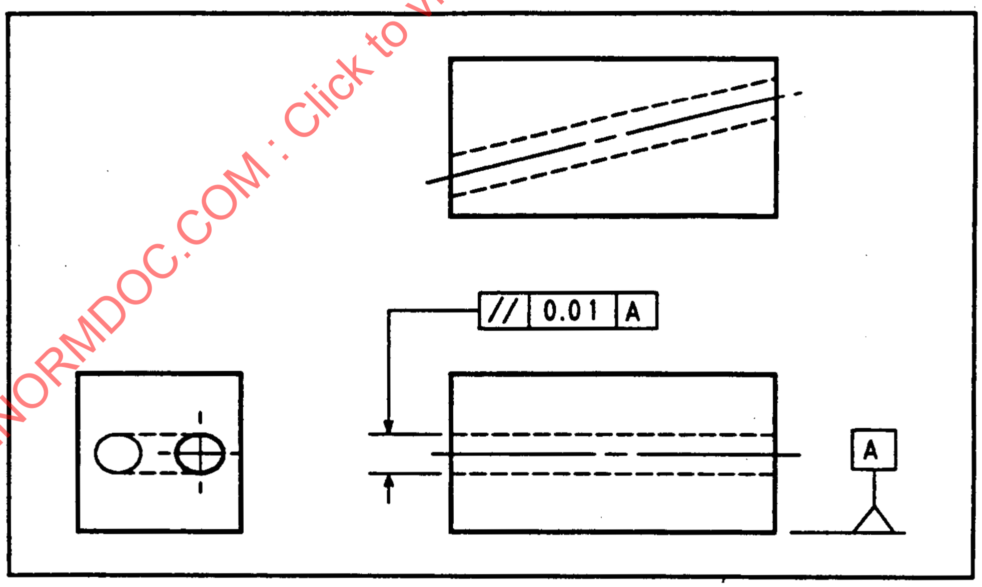


FIG. A-11 PLANAR PARALLELISM OF AN AXIS; PRIMARY DATUM PLANE; NO SECONDARY DATUM SPECIFIED

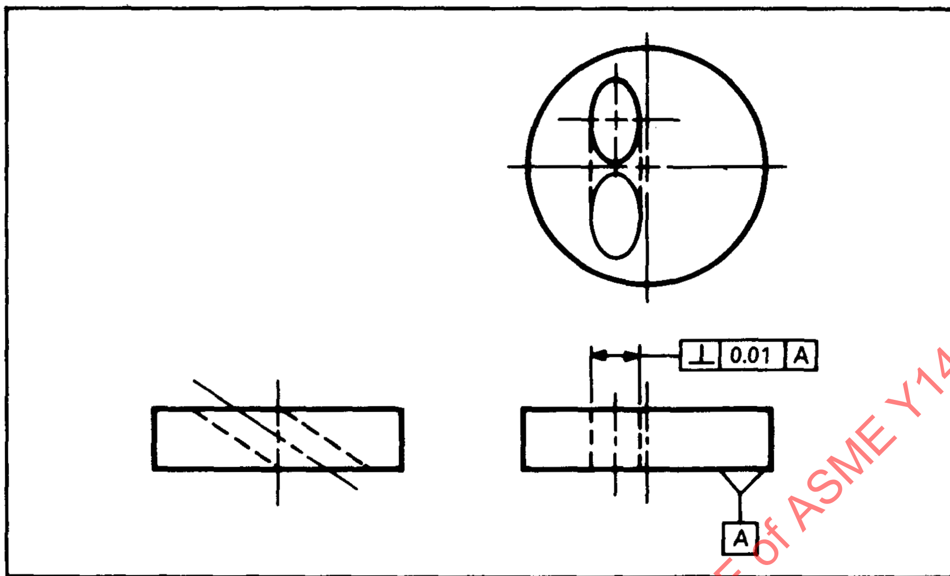


FIG. A-12a PLANAR PERPENDICULARITY OF AN AXIS; PRIMARY DATUM PLANE; NO SECONDARY DATUM SPECIFIED. LACK OF A SECONDARY DATUM MAY NOT PROVIDE ADEQUATE CONTROL

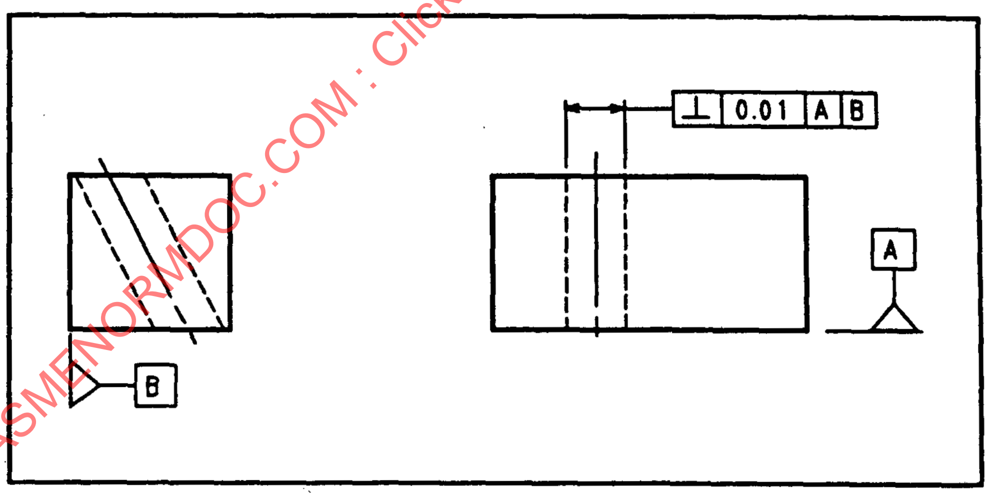


FIG. A-12b PLANAR PERPENDICULARITY OF AN AXIS; PRIMARY AND SECONDARY DATUM PLANES

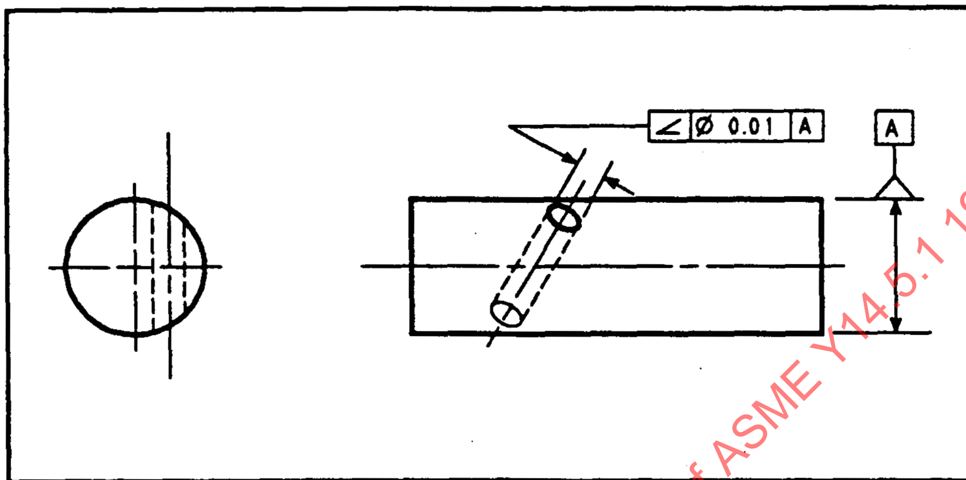


FIG. A-13a CYLINDRICAL ANGULARITY OF AN AXIS; PRIMARY DATUM AXIS; NO SECONDARY DATUM SPECIFIED

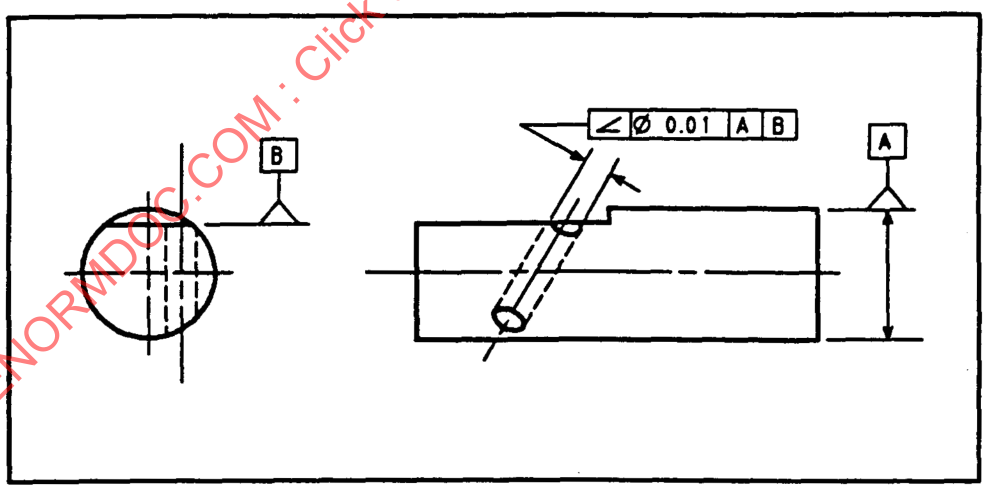


FIG. A-13b CYLINDRICAL ANGULARITY OF AN AXIS; PRIMARY DATUM AXIS; SECONDARY DATUM PLANE

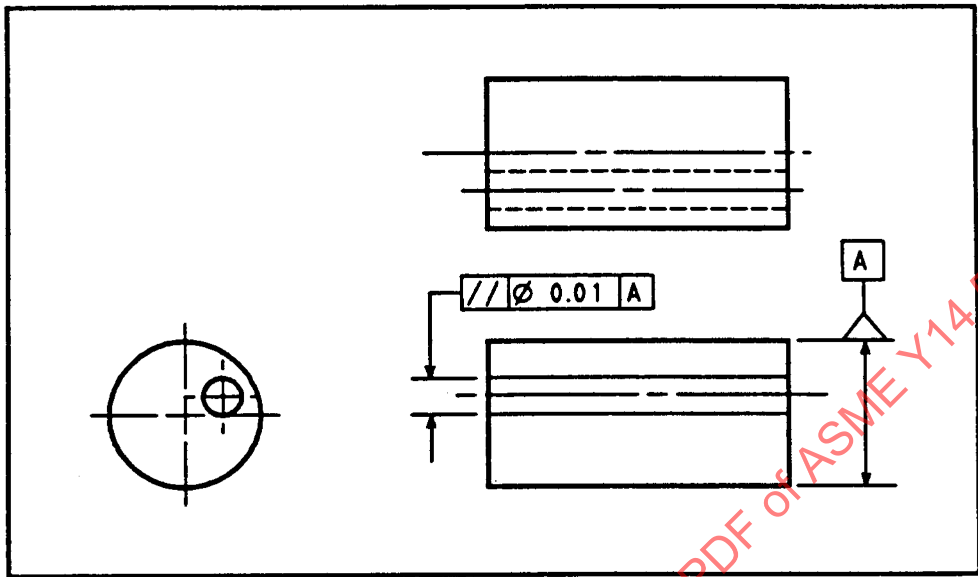


FIG. A-14 CYLINDRICAL PARALLELISM OF AN AXIS; PRIMARY DATUM AXIS; NO SECONDARY DATUM SPECIFIED

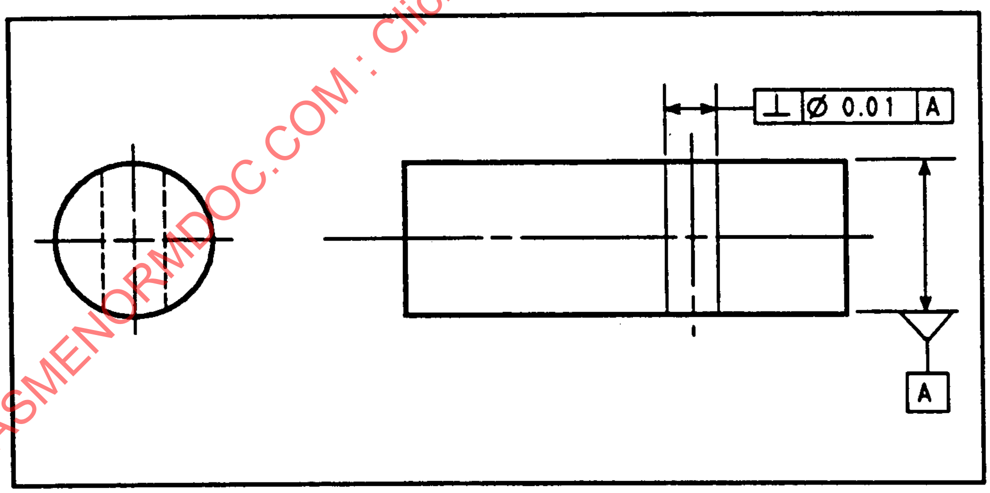


FIG. A-15a CYLINDRICAL PERPENDICULARITY OF AN AXIS; PRIMARY DATUM AXIS; NO SECONDARY DATUM SPECIFIED

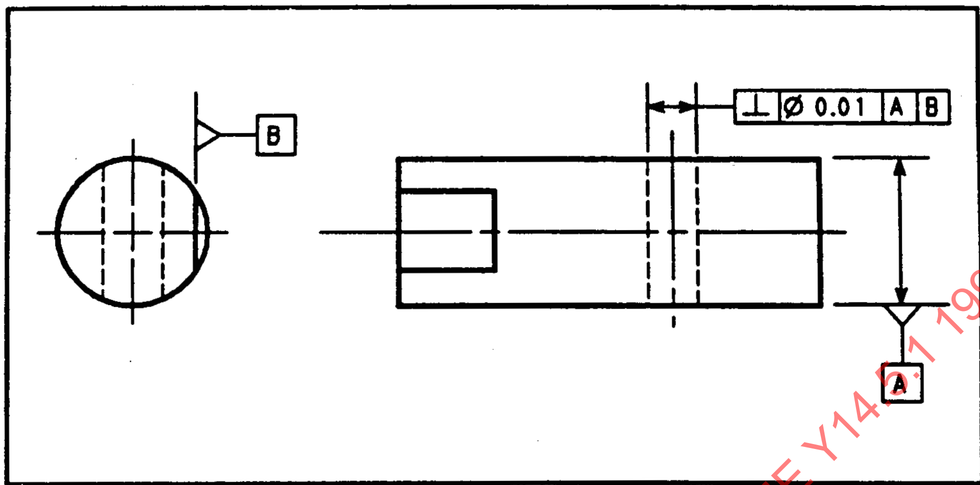


FIG. A-15b CYLINDRICAL PERPENDICULARITY OF AN AXIS; PRIMARY DATUM AXIS; AND SECONDARY DATA PLANE

A.3.4 Case 16. Figure A-16a shows a disk with a hole toleranced for angularity within a cylindrical zone with respect to a primary datum plane. Since no secondary datum is specified, this tolerance zone is free to rotate about the direction vector of the primary datum plane.

Figure A-16b shows a similar specification as in Fig. A-16a, with the difference being the application of a secondary datum. The specification shown requires that the axis of the tolerance zone be parallel to the secondary datum plane. Application of the secondary datum prevents rotation of the tolerance zone about the direction vector of the primary datum plane, hence fully restricts the orientation of the tolerance zone.

A.3.5 Case 17. Figure A-17 shows a part with a hole toleranced for parallelism within a cylindrical zone with respect to a primary datum plane. The specification requires that the axis of the tolerance zone be parallel to both the primary and secondary datum planes. If the secondary datum were not applied, then the resulting control would be the same as in case 11, and the use of the cylindrical zone would not impart additional control to the orientation of the hole.

A.3.6 Case 18. Figure A-18 shows a part with a hole toleranced for perpendicularity within a cylindrical zone with respect to a primary datum plane. The orientation of the tolerance zone is fully restricted; application of a secondary datum would not result in any additional control.

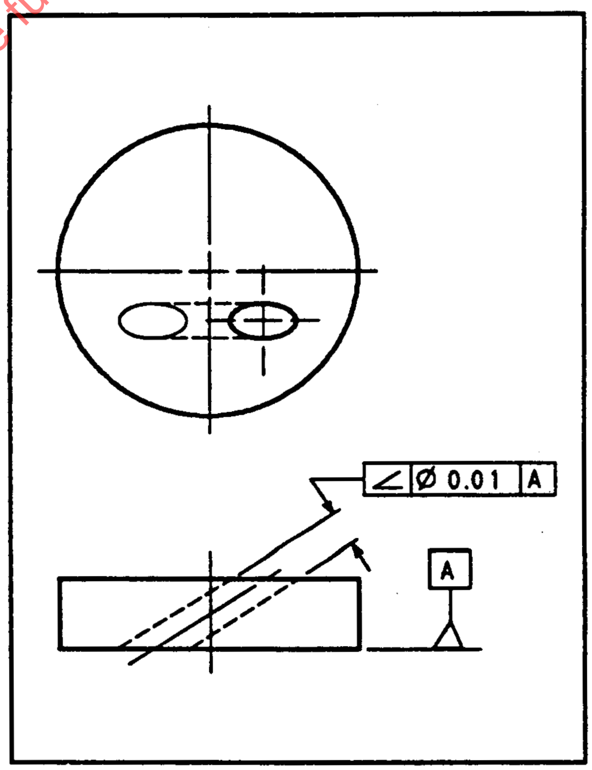


FIG. A-16a CYLINDRICAL ANGULARITY OF AN AXIS; PRIMARY DATUM PLANE; NO SECONDARY DATUM SPECIFIED